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Sleeping Beauty and De Nunc Updating

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SLEEPING BEAUTY AND *DE NUNC* UPDATING

A Dissertation Presented

by

NAMJOONG KIM

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ABSTRACT

SLEEPING BEAUTY AND *DE NUNC* UPDATING

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About a decade ago, Adam Elga introduced philosophers to an intriguing puzzle. In it, Sleeping Beauty, a perfectly rational agent, undergoes an experiment in which she becomes ignorant of what time it is. This situation is puzzling for two reasons: First, because there are two equally plausible views about how she will change her degree of belief given her situation and, second, because the traditional rules for updating degrees of belief don't seem to apply to this case.

In this dissertation, my goals are to settle the debate concerning this puzzle and to offer a new rule for updating some types of degrees of belief. Regarding the puzzle, I will defend a view called "the Lesser view," a view largely favorable to the Thirders' position in the traditional debate on the puzzle. Regarding the general rule for updating, I will present and defend a rule called "Shifted Jeffrey Conditionalization."

My discussions of the above view and rule will complement each other: On the one hand, I defend the Lesser view by making use of Shifted Jeffrey Conditionalization. On the other hand, I test Shifted Jeffrey Conditionalization by applying it to various credal transitions in the Sleeping Beauty problem and revise that rule in accordance with

the results of the test application. In the end, I will present and defend an updating rule called "General Shifted Jeffrey Conditionalization," which I suspect is the general rule for updating one's degrees of belief in so-called tensed propositions.

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INTRODUCTION

A. Problem

In his "Self-locating Belief and Sleeping Beauty," Elga (2000) presents an intriguing puzzle: On Sunday night, Sleeping Beauty (hereafter: SB) knows that she will go through the following experiment. On that night, she is put to sleep by a group of evil experimenters. Then, they toss a fair coin. Case 1: (*H*) The coin lands heads. In this case, the experimenters wake her up only once, on Monday. Case 2: (*T*) The coin lands tails. In this case, they wake her up twice, the first time on Monday and the second time on Tuesday. Between the two awakenings, they inject SB with a drug that erases her memory of the first awakening. In either case, one minute after she wakes up on Monday, she is told that it is Monday, and, when the experiment ends on Wednesday, she is awakened with her memory of the last awakening intact. The puzzle ends with two questions: When SB wakes up on Monday, what is her degree of belief in *H*? When she is told that it is Monday, what is her degree of belief in *H*?[1]

There have been two dominant answers to the first question in the literature. Halfers argue that the answer is 1/2 (Lewis 2001; Bradley 2003; Jenkins 2005): On Monday, she wakes up with Sunday as her last memory. Since she fully expected to wake up in that way, SB receives no new evidence relevant to *H* at that moment. Intuitively, a

[1] This version of the SB problem is closer to Lewis's (2001) version because the step of telling SB the day is omitted in Elga's original version (2000). In Chapter II, I will discuss Elga's (simpler) version, to focus on the first question, and in Chapter III and later, I will use Lewis's (more complex) version, to answer the second question.

rational agent changes her degree of belief (hereafter: credence) in a proposition *X* only when she receives new evidence relevant to *X*. SB's credence in *H* was 1/2 on Sunday night. Therefore, her credence in *H* on Monday is also 1/2. (Lewis 2001, p. 174.)

Thirders, on the other hand, contend that the answer is 1/3 (Elga 2000; Dorr 2002; Weintraub 2004). According to their view, when SB wakes up on Monday, she knows that she is waking up on either Monday or Tuesday but does not know which day it is. On the one hand, she assigns 1/2 to *H* conditional on the possibility that she is waking up on Monday. For remember that on Sunday night, she assigned 1/2 to *H* conditional on her waking up on Monday. On the other hand, she assigns 0 to *H* conditional on the possibility that she is waking up on Tuesday. This is because SB knows that if she is waking up on Tuesday, the coin has already landed on tails. It follows that her actual credence on Monday in *H* will be the weighted average between 1/2 and 0, where the weights come from her credences that she is waking up on Monday and that she is waking up on Tuesday. Since she cannot be sure that she is waking up on Monday, her rational credence in *H* is less than 1/2. If we take symmetry into consideration, it will be 1/3. (However, many philosophers doubt that symmetry can restrict an agent's credence in this way. For the purposes of my argument, the exact value of SB's credence on Monday in *H* is not important. The important element of the Thirders' view is that SB's credence in *H* is less than 1/2 when she wakes up on Monday.) (Elga 2000, pp. 144-145.)

On the second question, most Halfers and Thirders agree that when she learns that it is Monday, SB's credence in *H* increases, but they disagree about the precise value of the resulting credence in *H*. Many Halfers believe that it is 2/3, and virtually all Thirders believe that it is 1/2.

To compare their views, look at Table 1:

Table 1: The Change of SB's Credence in H

	Sunday night	Monday morning	told "Monday"
Halfers	1/2	1/2	2/3
Thirders	1/2	1/3	1/2

Ever since the publication of Elga's paper, philosophers have debated between these two options. So which side is right?

B. Goals

In this dissertation, I pursue two goals: First, I shall offer a solution to the SB problem; basically, I shall defend the Thirders' view *minus* any use of symmetry. Second, I shall develop a general rule for updating *de nunc* credences; in other words, I shall discuss a method for updating degrees of belief in tensed propositions.[2]

As the SB problem has attracted so many philosophers' attention, my pursuit of the first goal hardly requires any explanation. But why do I pursue the second? I do so because it offers the straightest solution to the SB problem.

To appreciate this point, think about these facts: We know SB's credence distribution on Sunday night fairly well. For instance, we know that her credence in *H* on that night is 1/2, we know that she knows then that it is Sunday, etc. Thus, finding her

[2] By "*de nunc* credence," I mean one's degree of belief in a tensed proposition. By "tensed proposition," I mean a proposition-like entity that may have different truth-values relative to time, but not relative to the individual. For instance, "it is raining now in Boston" may have different truth-values on Monday and Tuesday, but if it is true/false on Monday for someone, then it is true/false on Monday for everyone.

credence in H on Monday must be simply a matter of applying an appropriate updating rule to SB's credal transition from Sunday night to Monday morning. Therefore, if we have a correct rule for updating applicable to this transition, it will be easy, in principle, to calculate her credence in H on Monday.

Nevertheless, this approach has not been taken by many philosophers. Why? The traditional updating rule, called "Strict Conditionalization," states that an agent's credence at t in a proposition X is her conditional credence at $t' < t$ in X given E, where E is the totality of her observations during $(t', t]$. Unfortunately, this rule does not apply to SB's credal transition from Sunday night to Monday morning. SB learns (W) "SB wakes up today with the memory of Sunday as the last memory remembered" on Monday. However, on Sunday SB must have known (W') "SB woke up today with the memory of Saturday as the last memory remembered." Since W is logically incompatible with W', her conditional credence on Sunday night in H given W has no defined value. Therefore, Strict Conditionalization fails to provide a defined value for the former credence.

Hence, the most effective way to solve the SB problem is to apply an appropriate updating rule to SB's transition from Sunday night to Monday morning, but the Strict Conditionalization rule is inappropriate here. While many other philosophers have tried to solve the problem by appealing to other considerations for this difficulty (e.g. Lewis appeals to his Principal Principle to settle the debate (Lewis 2001), Kierland and Monton resort to the principle of minimizing inaccuracy of one's credence (Kierland and Monton 2005), etc.), I believe that finding the correct rule for *de nunc* updating is the best way to settle the debate.

In summary, I pursue two goals: Solving the SB problem and finding the general rule for *de nunc* updating. I have suggested that achieving the second goal will be the quickest way to achieve the first, but this does not mean that the second goal only has a derivative value. Indeed, it is the opposite: The SB problem is so interesting precisely because it reveals the fact that a new rule for updating is necessary to deal with *de nunc* credences properly.

C. Strategy

As I said above, I intend to develop a new rule for updating *de nunc* credences. To this end, I have developed the following criteria for an acceptable rule:

> **Solution** I want my updating rule to provide intuitive answers to the two questions of the SB problem.
>
> **Versatility** I want my updating rule to apply to as many types of updating situations as conceivable.
>
> **Coherence** I want my updating rule to provide coherent results.[3]

How can I develop a *de nunc* updating rule that satisfies these criteria? I start by reviewing the distinction between *updating* and *revision*, an established distinction in the literature of qualitative belief change.[4]

[3] Here, I am *not* using "coherent" in the technical sense that a rational agent's credence should not lead her to a Dutch book or money pump. Rather, I am using that word to mean freedom from incoherence, where incoherence is defined to be logical or conceptual inconsistencies. Bricker (ms.) says: “One way for a theory to be internally coherent is for it to be logically inconsistent, but I suppose there are other ways. Moreover, if a theory is unfaithful to the notions it aims to elucidate, be they notions of ordinary or scientific thought, that too is a form of incoherence.” (Bricker ms., p. 1.)

Revision An agent *revises* her beliefs because she has acquired better information about what the world is like.

Updating An agent *updates* her beliefs because she has noticed that the world has changed.

For instance, compare the following ways in which Jane comes to know that Barack Obama won the presidential election in 2008: First, it is 2009 and Obama has been president for some time, but Jane has *come to know* this fact today. Second, it is the morning after the day of the 2008 election, and Jane has come to know that he *became* the winner of the election last night. The former example is a case of revision, while the latter is a case of updating.

To see why this distinction is important, first look at this formulation of Strict Conditionalization: for any proposition X,

(1) $C_t(X)=\mathrm{C}_{t'}(X/E)$

where C_t and $C_{t'}$ are the agent's credence functions at t and $t'<t$, and E is the totality of her observation made during $(t',t]$. Now, note that (1) entails:

(2) $C_t(X/E)=\mathrm{C}_{t'}(X/E)$

[4] For a general theory of qualitative belief change (called "the AGM model"), see Gardenfors et. al. (1985). For the distinction made here, see Katsuno and Mendelzon (1992).

where *X* and *E* are genuine propositions (or proposition-like entities whose truth-values are fixed). This means that if an agent obeys Strict Conditionalization, she comes to preserve her past conditional credences. As Christensen points out (2000), this is a form of epistemic conservatism:

> The reasonableness of attractive instances of conditionalization seems to flow directly from the reasonableness of maintaining the relevant conditional degrees of belief. And these conditional degrees of belief are valuable because they reflect past learning experiences. (Christensen 2000, p. 354)

In this sense, epistemic conservatism is a good thing because one has worked hard to acquire valuable information about the world and to incorporate such information into one's belief state in the form of conditional credence.

But what if the world changes? Note that from an agent's present point of view, her past credence in *X* given E_i is the measurement of how probable it was that *X* was true *then* given that E_i was true *then*. In this sense, her past conditional credences are *outdated*. Consequently, epistemic conservatism is a disaster in this case: although the agent's past conditional credence in *X* given *E* was her degree of belief in *X*'s *then* truth given *E*'s *then* truth, she came to preserve it as her degree of belief in *X*'s truth *now* given *E*'s truth *now*. This is clearly unreasonable unless the agent has a reason to believe that the world is likely to stay in the same state as before. I see no reason to have such a belief about the world.

Note that this was not a problem in the traditional version of Strict Conditionalization, because that rule concerns only *de dicto* credences and evidence—the degrees of belief in propositions with fixed truth-values and in evidence whose content's truth-value is similarly fixed. Still, the following variant of Strict Conditionalization appears to be incorrect: for any *tensed* proposition X,

(3) $C_t(X)=C_{t'}(X/E)$

where $C_{t'}$ and C_t are B's credence functions at t and $t'<t$, and E is the totality of her observations during $(t',t]$. (3) implies

(4) $C_t(X/E)=C_{t'}(X/E)$

where X and E are tensed propositions. (4) is epistemologically dangerous for the reason explained above, which suggests that although Strict Conditionalization is a proper rule for the *revision* of *de dicto* credences, it is not a proper rule for the *updating* of *de nunc* credences.

Think about this: In the past, you had conditional credences about what *would* happen in a future time v given that you *would* observe E in v, and, from your present point of view, it might be v now.[5] Given this fact, if you know your present temporal location, then the following principle seems to capture a correct form of epistemic

[5] I am using "v" instead of "t" because I want to emphasize that the mentioned time is an inter"v"alized time, rather than a momentary one.

conservatism: Suppose that the agent B observes nothing during (t',t), for some moments t and $t'<t$. Then, for any proposition X,

(5) $C_t(X)=C_{t'}(X \text{ is true in } v/E \text{ is true in } v)$

where $C_{t'}$ and C_t are the agent's credence functions at t and t', E is the totality of the agent's observations made at t, and v is a temporal interval that B fully believes at t that she is in.[6]

I believe that (5) is often a correct rule for updating *de nunc* credences. For if an agent changes her *de nunc* credences obeying (5), then her resulting conditional credence in X given E will be equal to her previous conditional credence in X's truth in v given E's truth in v, *where v is her present temporal location*. For example, suppose that on Tuesday Jake learns that there will be a form of precipitation today and sets his credence in raining today to be equal to his conditional credence on Monday that it rains on Tuesday given that there is a form of precipitation on Tuesday. In a sense, he preserves his conditional judgment of how likely it is to rain on a day d given that there is a form of precipitation on d, where d is the same day referred to on Monday as "Tuesday" and referred to on Tuesday as "today." In this case, it is intuitive that Jake has to update in accordance with (5) because it is the best way to respect his past learning.

However, (5) is not general enough for our purpose, because it does not apply to a case of temporal uncertainty such as the SB problem. We need another candidate for the

[6] By "X is true in v," I mean that X is true at any moment in interval v. Likewise for "E is true in v." Plus, I assume that interval v is sufficiently narrow, but I do not try to provide a criterion for sufficient narrowness here.

general rule for *de nunc* updating. Consider this one: Suppose that an agent *B* observes nothing during (t',t) for some momentary times t, t' such that $t'<t$. Then, for any tensed proposition X,

$$(6)\ \ C_t(X)=\sum\nolimits_{i\in J}C_{t'}(X\text{ is true in }v_j/E\text{ is true in }v_j)C_t(\text{it is }v_j),$$

where $C_{t'}$ and C_t are *B*'s credence functions at t and $t'<t$, E is the totality of the *de nunc* content of the observation made at t, and $\{v_j\}_{j\in J}$ is a partition of temporal intervals such that *B* fully believes at t that she is in one of $\{v_j\}_{j\in J}$.

To me, (6) is a plausible generalization of (5). To see why, let $\{v_j\}_{1\leq j\leq n}$ be a partition of temporal intervals each of which *B* thinks at t to be possibly her then temporal location. By (5),

$$C_t(X)\text{ would be }\begin{cases} C_n(X\text{ is true in }v_1/E\text{ is true in }v_1)\text{ if }B\text{ were sure at }t\text{ that it is }v_1,\\ C_n(X\text{ is true in }v_2/E\text{ is true in }v_2)\text{ if }B\text{ were sure at }t\text{ that it is }v_2,\\ \qquad\ldots\\ C_n(X\text{ is true in }v_n/E\text{ is true in }v_n)\text{ if }B\text{ were sure at }t\text{ that it is }v_n,\end{cases}$$

Since *B* does not know what time it is, it appears to be natural to take the weighted average of the above values, with the weights coming from *B*'s credence at t that it is v_j. Thus, (6).

Equipped with (6), we are ready to answer the first question of the SB problem: Let s be SB's last conscious moment on Sunday, m be the moment of wakeup on Monday, and m+ be one minute after m when SB is told that it is Monday. Clearly, SB does not make any observation during (s,m). By (6),

(7) $C_m(H)=$

$C_s(H \text{ is true on Monday}/W \text{ is true on Monday})C_m(\text{it is Monday})+$

$C_s(H \text{ is true on Tuesday}/W \text{ is true on Tuesday})C_m(\text{it is Tuesday}).$

Since H is a genuine proposition and its truth-value is insensitive to time,

(8) $C_m(H)=$

$C_s(H/W \text{ is true on Monday})C_m(\text{it is Monday})+$

$C_s(H/W \text{ is true on Tuesday})C_m(\text{it is Tuesday}).$

Since SB fully knew on Sunday that if she wakes up on Tuesday, then H is false,

(9) $C_m(H)=C_s(H/W \text{ is true on Monday})C_m(\text{it is Monday}).$

Since she fully expected on Sunday to wake up on Monday,

(10) $C_m(H)=C_s(H)C_m(\text{it is Monday}).$

Since her credence at s in H was 1/2 and she cannot be sure at m that it is Monday,

(11) $C_m(H)=1/2C_m(\text{it is Monday})<1/2.$

This is a result favorable to the Thirder view and incompatible with the Halfer view.

Thus, (6) is not only a plausible principle for updating *de nunc* credences, but also it provides the SB problem with a *solution* that is largely favorable to the Thirder view. This means that it satisfies one of my criteria for an acceptable principle for *de nunc* updating.[7]

However, there are two reasons to suspect that (6) is not the end of the story. First, it follows from (6) that SB's credence in *H* does not change from the moment of wakeup on Monday to that of being told that it is Monday, but we have a reason to consider this to be an *incoherent* result. To see the reason, first think about the following instance of (5) (which is a special case of (6)): At *m*+, SB knows that it is Monday. Thus,

(12) $C_{m+}(H)=C_m(H \text{ is true on Monday}/MON \text{ is true on Monday})$,

where *MON* is the tensed proposition that it is Monday. Since *H* has a fixed truth-value and she knows at *m* that *MON* is, of course, true on Monday,

(13) $C_{m+}(H)=C_m(H)<1/2$.

This result does not cohere with

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[7] Of course, this result may not be satisfactory to the Halfers. But I am focusing here on the facts that (i) the provided solution is attractive to a large group of philosophers and (ii) it comes with an argument that might be plausible even to some philosophers who initially opposed its conclusion.

$$(14) \qquad C_m(H/MON)=1/2,$$

which is provable from the very (6).[8]

To understand why, think about this matter from SB's point of view when she is told that it is Monday. Previously, her credence in *H* was 1/2 given *MON*, and, by learning that it is Monday now, she also learns that it was previously Monday. Intuitively, her credence in *H* must increase back to 1/2 by this learning.

Second, (6) will not apply to a credal transition from t' to t if the agent makes any observation during (t',t). For instance, SB experiences *W* when she wakes up on Monday; thus, she makes a seemingly important observation between *s* (=the night on Sunday) and *m*+ (=one minute after her wakeup on Monday). Although we can apply (6) to her credal transition from *s* to *m* and to her credal transition from *m* to *m*+ (the second application is seen in the problem discussed in the last paragraph), it would be better if we could calculate SB's credence at *m*+ in *H* all at once from her credence distribution on Sunday night. In this sense, (6) does not satisfy the criterion of *versatility*.

In sum, while I have a promising prototype for the general rule for *de nunc* updating, it does not perfectly satisfy the three aforementioned criteria. My goal in this dissertation is to find a general rule for *de nunc* updating which is fully *versatile* and *coherent*, and which provides a fully intuitive *solution* for the SB problem.

D. Contents

This dissertation consists of six chapters:

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[8] Hint: Show first that $C_m(H\&MON)=C_m(T\&MON)$. Since $C_m(MON)=C_m(H\&MON)+C_m(T\&MON)$, $C_m(H/MON)=C_m(H\&MON)/C_m(MON)=C_m(H\&MON)/2C_m(H\&MON)=1/2$. (See Chapter II for a full proof.)

Chapter I. Introduction

Chapter II. Updating with a Single Observation

Chapter III. Updating with a Sequence of Observations

Chapter IV. Updating with *De Priori* Information

Chapter V. Satisfaction of Desiderata

Chapter VI. Conclusion

The titles of Chapters I and VI are self-explanatory. Roughly, the main body of the dissertation consists of three parts, each devoted to one of my criteria: (i) In Chapter II, I discuss a relatively simple principle for *de nunc* updating, to *solve* the SB problem. (ii) In Chapters III and IV, I generalize that simple principle into more *versatile* principles. (iii) In Chapter V, I prove that the most general principle has several properties that we can regard as forms of *coherence*. I provide more details below.

In Chapter II, I will discuss how a rational agent changes her credence in a tensed proposition from t to t', assuming that she receives no evidence during (t',t). I will present and defend the following principle for updating *de nunc* credences, which I call “Shifted Jeffrey Conditionalization" or “SJC": Let X be any tensed proposition. Then, roughly,

(15) $$C_t(X)=\sum_{i\in I, j\in J} C_{t'}(X \text{ is true in } v_j / E_j \text{ is true in } v_j)C_t(E_j \text{ is true\&it is } v_j),$$

where $C_{t'}$ and C_t are B's credence functions at t and t', $\{E_i\}_{i\in I}$ is a partition whose member represents an observation she might be making at t, and $\{v_j\}_{j\in J}$ is a partition whose member represents a temporal interval that she might be in at t.[9]

Since (6) is a special case of SJC (or (15)), SJC suffers from the two problems I discussed earlier. Figure 1 is a diagram showing how SB's credence in H changes in accordance with SJC:

$$\begin{array}{ccccc}
C_s(H) & & C_m(H) & & C_{m+}(H) \\
1/2 & & \xrightarrow{\qquad\text{N/A}\qquad} & & ? \\
1/2 & \xrightarrow{W} & x & \xrightarrow{MON} & x
\end{array}$$

Figure 1: <u>Update in Accordance with SJC</u>. Not as versatile as wanted. Some incoherent result.

where x is some value less than 1/2. First, SJC does not apply to the all-at-once updating from s to m, which makes it not as versatile as we want. Second, SJC yields the counter-intuitive result that her credence in H does not increase back to 1/2.

In Chapter III, I discuss how an agent can change her credence in a tensed proposition from t' to t, assuming that she makes a finite sequence of observations during $(t,t]$. I will present a rule governing this type of updating, which I call "Sequential Shifted Jeffrey Conditionalization" or "SSJC." I cannot provide a proper formulation of this rule here; simply, we do not have the necessary formal and conceptual resources yet. Instead,

[9] Note that (15) is more general than (6) in that (15) incorporates uncertainty about what observation was made as well as uncertainty about what time it is.

I provide its instance involving how SB's credence in *H* changes from the night on Sunday (=*s*) to when she is told that it is Monday (=*m*+):

$$(16)\quad C_{m+}(H) = \begin{pmatrix} C_s(H \text{ is true on Monday}/ \\ \text{(a)}\, W \text{ is true on Monday} \,\& \\ \text{(b)}\, MON \text{ is true on Monday} \end{pmatrix} \times \begin{pmatrix} C_{m+}(\\ \text{(c) previously, } W \text{ was true and it was Monday} \,\& \\ \text{(d) presently, } MON \text{ is true and it is Monday} \end{pmatrix}.$$

SB makes an observation twice during (*s*,*m*], the first time *W* and the second time *MON*. In other words, she makes a sequence of observations <*W*,*MON*> during (*s*,*m*]. Here is the core idea of (16): To find SB’s rational credence in *H* given this sequence of observations, we need to specify, for each element of this sequence, the time of her observing it, as you see in (c) and (d), and figure out her prior credence in *H* given that each element of the sequence is true at those specified times, as you see in (a) and (b). Hopefully, the reader will see how to generalize this idea into a formal rule for updating. Unfortunately, SSJC is inconsistent. Figure 2 is a diagram showing how SB's credence in *H* changes in accordance with SSJC:

$$\begin{array}{ccccc} C_s(H) & & C_m(H) & & C_{m+}(H) \\ 1/2 & & \xrightarrow{\quad\langle W,MON\rangle\quad} & & 1/2 \\ 1/2 & \xrightarrow{W} & x & \xrightarrow{MON} & x \end{array}$$

Figure 2: Update in Accordance with SSJC. Versatile but inconsistent because $x \neq 1/2$.

where *x* is some value less than 1/2. As you see above, SSJC provides a different result depending upon whether we apply it to the transition from *s* to *m*+ or to the transition from s to m and then to that from m to m+. This makes it an unacceptable rule (unless restricted by a suitable proviso).

In Chapter IV, I discuss how a rational agent changes her credence in a tensed proposition from t' to t, assuming that her observation may include information about her temporal location at t' or an earlier moment. The updating rule presented in this chapter will be the most general updating rule discussed in this dissertation. I will call this rule "General Shifted Jeffrey Conditionalization" or "GSJC." Again, I do not try to present the rule here. Instead, I discuss how it solves the problem that SB's credence in *H* does not change at m+ although her then evidence *MON* is intuitively relevant to *H*. Under several assumptions, we can derive the following claim from GSJC:

$$(17)\ C_{m+}(H) = \begin{pmatrix} C_m(H \text{ is true on Monday}/ \\ \text{(a)} \mathit{MON} \text{ is true on Monday} \,\& \\ \text{(b) presently, it is Monday} \end{pmatrix} \times \begin{pmatrix} C_{m+}(\\ \text{(c) presently, } \mathit{MON} \text{ is true and it is Monday} \,\& \\ \text{(d) previously, it was Monday} \end{pmatrix}.$$

In this updating process, *MON* is the only thing that SB has learned during (m,m+]. In a trivial sense, we can say that <*MON*> is the sequence of observation that she has made during (m,m+].

Here is the core idea in (17): To find her rational credence in *H* given this sequence of an observation, we need to specify the time of her observing *W* and figure out her prior credence in *H* given *W*'s truth at the specified time (which happens to be Monday), as you see in (a) and (c). But that's not all. We also need to specify what time it was *before* she observed *W* and figure out her prior credence in *H* given *W*'s truth on

Monday *plus* the prior time's being Monday, as you see in (b) and (d). I will call such information about what time it was at the prior time "*de priori* information."[10]

Let us see how this modification provides a better model for how SB's credence in H changes. According to GSJC, SB's credence in H changes as in Figure 3:

$$
\begin{array}{ccccc}
C_s(H) & & C_m(H) & & C_{m+}(H) \\
1/2 & & \xrightarrow{\quad\langle W,MON\rangle\quad} & & 1/2 \\
1/2 & \xrightarrow{W} & x & \xrightarrow{MON} & 1/2
\end{array}
$$

Figure 3: Update in accordance with GSJC. Versatile, coherent, providing results compliant with the popular Thirder view.

where x is some value less than 1/2. As Figure 3 demonstrates, GSJC provides a coherent and intuitive model for the change of SB's credence in H. This model also complies with the view of Thirders, regarding both questions asked earlier.

In Chapter V, I argue that GSJC has several properties desirable for any cogent rule for updating. In particular, I will show that (i) GSJC can be regarded as a binary relation between the given agent's credence functions (at different times) and (ii) as such, GSJC is transitive, if all of the agent's credence functions are synchronically coherent. See Figure 4:

[10] I owe this term to Gareth Matthews.

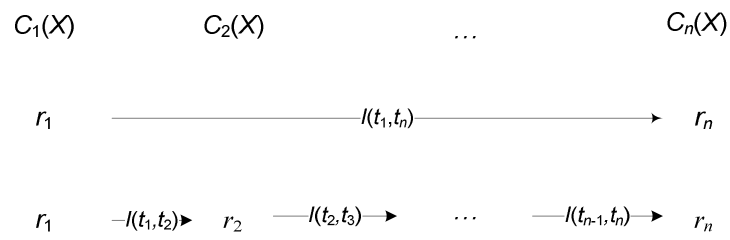

Figure 4: Transitivity of GSJC. Here, I(t,t′) is information observed during (t,t′].

Put another way, GSJC provides the same result whether you update all at once from t_1 to t_n or step-by-step from t_1 to t_2, t_2 to t_3, … to t_n.

In Chapter VI, I discuss (i) how to modify GSJC into a general rule for *de se* updating, (ii) how to overcome a potential problem of GSJC, and (iii) whether there is any comparably plausible but simpler rule for updating. I conclude that GSJC is likely to be identical or very close to the general rule for *de se* updating.

Through these discussions, I will defend my view that GSJC is the rational rule for updating *de nunc* credences. I will argue that GSJC provides not only an ideal solution for the Sleeping Beauty problem but also a versatile and coherent general rule for *de nunc* updating.

CHAPTER II

UPDATING WITH A SINGLE OBSERVATION[11]

A. Introduction

SB problem 0. Suppose that Sleeping Beauty (hereafter: SB), a paragon of probabilistic rationality, knows the following facts on Sunday: A group of evil experimenters will put her to sleep on that day. Next, they will toss a fair coin. Case 1: (*H*) The coin lands heads. In this case, the experimenters will wake SB only on Monday. Case 2: (*T*) The coin lands tails. In this case, they will wake SB for the first time on Monday, inject her with a drug that erases her memory of Monday, and then wake her for the second time on Tuesday. In either case, the experiment is over on Wednesday.

For brevity, let *s* be the last moment on Sunday at which SB is conscious and let *m* be the moment of waking up on Monday. Accordingly, let C_m and C_s be her credence functions at *m* and *s*. The question is: "What is SB's credence at *m* in *H*?" There have been two dominant answers: According to the Thirder view, $C_m(H)=1/3$ (Elga 2000). According to the Halfer view, $C_m(H)=1/2$ (Lewis 2001). Both views have good arguments in their favor.

Halfers contend: Let *W* be that SB wakes up today with the memory of Sunday as the last memory. When SB was put to sleep on Sunday, she fully expected to receive *W* as her next evidence. Furthermore, that she has awakened today with such and such

[11] This chapter is identical to my paper published in Synthese (Kim 2009) except for several changes of notation and correction of typos.

memory seems irrelevant to whether the coin lands heads or tails. Hence, *W* is neither new nor relevant to *H*. But the thesis below derives from the standard rule for credence updating:

(1) If no new evidence relevant to *X* is received,
then it is irrational for an agent to change her credence in *X*.

Thus, SB doesn't change her credence in *H* after waking up on Monday. It is not controversial that SB assigns the credence of 1/2 to *H* on Sunday night. Therefore, she assigns 1/2 to *H* after waking up on Monday. (Lewis 2001, 174.)

Thirders argue: Let *MON* be that it is Monday, and *TUE* be that it is Tuesday. Then, we can define H_1, T_1, and T_2 as below:

H_1: *H&MON*

T_1: *T&MON*

T_2: *T&TUE*

Obviously, these exhaust the possibilities open to SB when she wakes up on Monday. On the one hand, suppose that SB was immediately told that it's Monday after waking up on Monday. In this scenario, she would assign the same credence of 1/2 to *H* and *T*. Hence, $C_m(H/MON)=1/2=C_m(T/MON)$. It follows that

(2) $C_m(H_1)=C_m(T_1)$.

On the other hand, assume that SB was told immediately after waking up on Monday that the coin landed on tails. The evidence she receives on Monday, in this scenario, is compatible with either *MON* or *TUE*; for, if the coin lands tails, she wakes up both on Monday and Tuesday. By a principle of indifference, it seems rational to assign the same credence to *MON* and *TUE*, given *T*; formally, $C_m(MON/T)=C_m(TUE/T)$. It follows that

(3) $C_m(T_1)=C_m(T_2)$.

In sum, $C_m(H_1)=C_m(T_1)=C_m(T_2)=1/3$. But H_1 is the only possibility in which the coin lands heads. Hence, $C_m(H)=1/3$. (Elga 2000, 143-144.)

Which side made a mistake? Theses (1) and (2) contradict each other.[12] Hence, Halfers, who accept (1), are bound to reject (2), and Thirders, who accept (2), are committed to the rejection of (1). Thus, each side complains that the other's argument is factually incorrect. The problem is that neither side has been able to *explain* why the other side's key premise is wrong. For this reason, the debate has continued. I suspect, however, that both sides have missed an important point. We know quite well SB's belief state on Sunday night; on that night, her credence in *H* was 1/2, and she knew how the experiment would proceed. *W* is the only evidence she acquires until she wakes up next morning. Isn't it then simply a matter of applying Strict Conditionalization (hereafter: SC), the traditional principle for updating credences, to SB's credal transition from *s* to *m*?

[12] Suppose theses (1) and (2). Hence, $C_m(H)=C_s(H)=1/2$ and $C_m(H_1)=C_m(T_1)$. Unless $C_m(T_2)=0$, $C_m(H_1)<C_m(T_1)+C_m(T_2)$. Since H_1 exhausts the *H*-possibility and T_1 and T_2 exhaust the *T*-possibilities, $C_m(H)<C_m(T)$. Therefore, $1/2=C_m(H)<1/2$. Done.

Unfortunately, the problem is not that simple. It is an instance of SC that $C_m(H)=C_s(H/W)$. However, SB fully knew on Sunday night that she did *not* wake up on that day with the memory of Sunday as the last memory, and so $C_s(H/W)$ is undefined.

Nevertheless, I believe that there exists an updating principle applicable to SB's credal transition from *s* to *m*. My first goal in this chapter is to find an updating rule for when the domains of an agent's credence functions include *tensed propositions* or proposition-like entities whose truth-values are possibly different depending upon the time of evaluation. I will call this new updating principle "Shifted Jeffrey Conditionalization" (hereafter: SJC). I will argue that because *W* is a tensed proposition, SB has to use SJC, not SC, for her credal transition from *s* to *m*.

My second goal in this chapter is to explore the ramifications of this new updating rule concerning the SB problem. I shall make three claims: First, thesis (1) is disprovable under SJC. Second, thesis (2) is provable under SJC. Third, thesis (3) is neither provable nor disprovable in any obvious way.

If these claims are true, who wins in the Sleeping Beauty debate? On the one hand, Halfers are clearly not the winners. For their argument is unsound if thesis (1) is false and SB's credence at *m* in *H* is less than 1/2 if thesis (2) is true. On the other hand, this does not necessarily mean a victory for the Thirders. For thesis (3) is an essential element of their view but SJC does not obviously support it.

Consequently, I partially accept the Thirder view but take a more lenient position: I will argue that SB's credence in *H* is less than 1/2 when she wakes up, but I will remain silent about what the value should be. Call this "the Lesser view." In this chapter, I will defend it.

I will proceed in the following order: In Section B, I will clarify my assumptions and terminology and review traditional updating principles. In Section C, I will argue that those updating principles do not work for beliefs and evidence whose truth-values are different relative to time and/or individual. In Section D, I will present an alternative updating principle, SJC, for such beliefs and evidence. In Section E, I will defend SJC by extending Gaifman's influential view of expert principles. Finally, in Section F, I will apply SJC to the SB problem. As a result, the Halfers' thesis (1) will be criticized and the Thirders' thesis (2) will be defended.

B. Background

In this section, I will clarify my assumptions and terminology about beliefs, contents, and credences.

First, belief is a relation between an agent and a proposition-like entity. For example, consider Jane's belief that (*C*) Caesar crossed the Rubicon in 49 BC. According to my assumption, this belief is a relation between Jane and *C*.

Second, the truth-values of some beliefs remain the same whoever has them or whenever they are had. Jane's belief in the above paragraph is a good example. Let's call such a belief a "*de dicto* belief" and its content a "(genuine) proposition." I consider such a belief to be purely about which possible world the agent is located in.

Third, the truth-values of some beliefs are different relative to times and/or individuals. For example, consider Jane's belief expressed by "I am 15 years old." The content of this belief will be true of anyone who is 15 years old, but won't be true of anybody who is younger or older. Let's call such a belief an "irreducibly *de se* belief" and its content an "irreducibly centered proposition." I consider a belief of this type to be at least partially about what time it is and/or who the agent is.

Fourth, I will call any *de dicto* or irreducibly *de se* belief simply "a *de se* belief" and its content "a centered proposition."[13]

Fifth, belief is not all-or-nothing but comes in degrees. Degrees of belief, called "credences," are probabilities in that they satisfy Kolmogorov's three axioms: Non-Negativity, Normality, and Additivity.

Sixth, I will call the degree of a *de dicto* belief a "*de dicto* credence," that of an irreducibly *de se* belief an "irreducibly *de se* credence," and that of a *de se* belief a "*de se* credence."

So far, I have clarified my assumptions and terminology. Now, to the question: "What is the correct rule for updating *de se* credences?" To answer, it is a good idea to review the traditional rules for updating *de dicto* credences.

First, how is a rational agent supposed to update her *de dicto* credences given *certain* evidence? Consider the strongest proposition E such that a rational agent B becomes certain at t_{n+1} of E, as a result of her experience at t_{n+1}.[14] Then, B should update by Strict Conditionalization: (SC) for any proposition X, $C_{n+1}(X)=C_n(X/E)=_{\mathrm{df}}$ $C_n(X\&E)/C_n(E)$, where $C_n(E)\geq 0$. To see how SC works, consider this example: **Example**

---

[13] I am following Lewis (1979) in defining *de se* belief in this way: "I say that *all* belief is 'self-locating belief.' Belief *de dicto* is self-locating belief with respect to a logical space; belief irreducibly *de se* is self-locating belief at least partly with respect to ordinary time and space, or with respect to the population" (Lewis 1979, 522).

[14] For simplicity, I will assume that the given agent makes observations at only a countable number of moments in her life. Let's call them "epistemic moments." From now on, I will use "t_α" to refer to a series of epistemic moments, where α indicates the *order* and *contiguity* of those moments. Hence, for any $m,n\in N$, t_m is later than t_n iff $m>n$, and for any $n\in N$, t_{n+1} is the epistemic moment next to t_n. In addition, I will use "C_α" to refer to the given agent's credence function at t_α.

1. Jane's previous conditional credence in a coin's landing heads was 3/4 given that it is tossed. After she receives the evidence that it was tossed, her present unconditional credence in the coin's landing heads becomes 3/4.

Second, how does a rational agent update her *de dicto* credences if no proposition meets the condition satisfied by E in the last paragraph? Suppose that $\{E_i\}_{i\in I}$ is a partition such that for any $i\in I$, an agent B's credence in E_i is directly set by her experience at t_{n+1}. I will call each E_i "(B's) observation proposition at t_{n+1}." Let $\{E_i\}_{i\in J}$ be a subset of this partition (so $J\subseteq I$) such that $C_{n+1}(E_i)>0$ for any $i\in J$. If also $C_n(E_i)>0$ for any $i\in J$, then we call $\{E_i\}_{i\in J}$ "(B's) observation partition at t_{n+1}." In such a case, Richard Jeffrey suggests that B should update her *de dicto* credences by Jeffrey Conditionalization:

(JC) for any proposition X, $C_{n+1}(X)=\sum_{i\in J} C_n(X/E_i)C_{n+1}(E_i)$ (Jeffrey 1990, pp. 164-83).

To see how JC works, consider this example: **Example 2**. At 2:00 PM, Jane is looking at a piece of vegetable under a dim light, uncertain whether it is green or violet. Hence, she is uncertain about which of G ("this piece of vegetable is green") and V ("this piece of vegetable is violet") is true. Still, her experience somehow influences her credences in G and V; consequently, her credences at 2:00 PM in G and V are 0.3 and 0.7. Then, what should her credence be in C ("it is a piece of cabbage")? Her conditional credence at 1:59 PM in C was 0.6 given G and 0.2 given V. By JC, $C_{2:00\text{ PM}}(C)=C_{1:59\text{ PM}}(C/G)C_{2:00\text{ PM}}(G)+C_{1:59\text{ PM}}(C/V)C_{2:00\text{ PM}}(V)=0.32$.

If, as I have demonstrated, SC or JC is the rule for updating *de dicto* credences, what then is the rule for updating *de se* credences? According to David Lewis, we can easily find a candidate for such a rule: just replace "*de dicto*" with "*de se*" and "proposition" with "centered proposition" in SC (Lewis 1979, 534). Another candidate can be found by carrying out the same replacement in JC.

However, I believe that the *de se* versions of SC and JC are incorrect. For they have a common problem, which I discuss in the next section.

C. <u>A Problem of the *De Se* Versions of SC and JC</u>

I want to show that the *de se* versions of both SC and JC are untenable. However, criticizing them will be a tedious job if done one by one. A more efficient method will be to first find a thesis common to the two principles and, second, show that this common thesis has a fatal problem. Is there such a thesis?

According to Richard Jeffrey (1984, p. 135), we can easily prove this:

> (C) Suppose that both C_n and C_{n+1} satisfy Kolmogorov's axioms. Let E be the agent's total evidence at t_{n+1}. Then, (a) for any proposition X, $C_{n+1}(X)=C_n(X/E)$ iff (b) $C_{n+1}(E)=1$ and (c) for any proposition X, $C_{n+1}(X/E)=C_n(X/E)$.

In other words, an agent is a strict conditionalizer iff she certainly believes her total evidence, and its probabilistic relevance to any other belief is unchanged by updating. Following Jeffrey's terminology, let's call condition (c) "Rigidity." We can also prove this broader claim (Jeffrey 1984, p. 136):

> (K) Suppose that both C_n and C_{n+1} satisfy Kolmogorov's axioms. Let $\{E_i\}_{i\in I}$ be the agent's observation partition at t_{n+1}. Then, (d) for any proposition X,

$$C_{n+1}(X) = \sum_{i\in I} C_n(X/E_i)C_{n+1}(E_i) \text{ iff (e) for any proposition } X \text{ and } i\in I,$$

$$C_{n+1}(X/E_i)=C_n(X/E_i).$$

In other words, an agent is a Jeffrey conditionalizer iff her old and new credence functions satisfy Rigidity. According to (C) and (K), the truth of Rigidity regarding each member of her observation partition is a common necessary condition of SC and JC.[15] Now, replace "proposition" with "centered proposition" in (C) and (K); still, they are provable claims. Let's call the results of this replacement within (a), (d), and (c)/(e) "the *de se* versions of SC, JC, and Rigidity." Then, the *de se* version of Rigidity is the common necessary condition of those of SC and JC.

But Rigidity has a fatal flaw. It conflicts with the logic of *de se* beliefs,[16] in that a *probabilistic* updating pattern it supports often leads to *deductively* invalid reasoning. Think about this example: **Example 3**. Let R be the centered proposition expressed by "it is raining now," and P be the one expressed by "some form of precipitation is occurring now." Assume that $\{P, \text{not-}P\}$ is Jake's observation partition at 2:00 PM. For our purpose, it is best to discuss the present example from the first-person point of view; hence, we let C_{prev} be Jake's credence function at 1:59 PM and C_{now} be his credence function at 2:00

[15] If the agent has certain total evidence E, then E is the sole member of her observation partition.

[16] Meacham (2008) provides a simpler argument against the *de se* version of SC: Once a *de se* SC-er becomes certain that it is 9:00 AM, she cannot abandon that belief at 9:01 AM. Since such abandonment occurs too often, the *de se* version of SC cannot be true. This argument seems to be sound, but I have two reasons to look for an alternative. First, Meacham's argument does not show the incorrectness of the *de se* version of JC. Second, his argument cannot defeat the *de se* version of SC formulated in terms of primitive conditional credences. For the problem raised by Meacham is a form of the zero-denominator problem, which can be avoided if we define unconditional credence in terms of conditional credence rather than doing the opposite (Hajek 2003).

PM. Additionally, assume that $C_{prev}(P)=0.5$ and $C_{now}(P)>0$. Obviously, we can derive this fact from the *de se* version of Rigidity:

(4) If $C_{prev}(R/P)\approx 1$, $C_{now}(R/P)\approx 1$.[17]

From the suppositions, we can easily show that (i) if $C_{prev}(P\supset R)\approx 1$, then $C_{prev}(R/P)\approx 1$ and (ii) if $C_{now}(R/P)\approx 1$, then $C_{now}(P\supset R)\approx 1$.[18] Hence, this is true:

(5) If $C_{prev}(P\supset R)\approx 1$, then $C_{now}(P\supset R) \approx 1$.

This means that the following normative sentence is true of Jake at 2:00 PM:

(6) If I strongly believed $P\supset R$ previously,

I must strongly believe $P\supset R$ now.

Practically, following (6) amounts to doing this type of reasoning:

(7) It was previously the case that, if *P*, then *R*.

Therefore, it is now the case that, if *P*, then *R*.

---

[17] Here, "≈" is a synonym of "is almost identical to."

[18] Here, "⊃" is the material implication connective. To prove (i), suppose that $C_{prev}(P\supset R)\approx 1$. We know that $C_{prev}(P\supset R)=C_{prev}(\text{not-}(P\&\text{not-}R))=1-(1-C_{prev}(R/P))C_{prev}(P)$. By the assumption that $C_{prev}(P)=0.5$, $C_{prev}(P\supset R)=1/2(1+C_{prev}(R/P))$. Thus, $C_{prev}(R/P)=2C_{prev}(P\supset R)-1$. By supposition, $C_{prev}(R/P) \approx 1$. To prove (ii), suppose that $C_{now}(R/P) \approx 1$. Since $C_{now}(P\supset R)=1-(1-C_{now}(R/P))C_{now}(P)$, $C_{now}(P\supset R)\approx 1$. Done.

However, this argument form is invalid. For it is surely possible that the premise is true but the conclusion is false: It was previously raining, and it is now snowing. Hence, the *de se* version of Rigidity may lead to invalid reasoning.[19] This is a good reason to abandon it.

Since the *de se* version of Rigidity is a common necessary condition of SC and JC, we should reject both.

D. Shifted Jeffrey Conditionalization

If neither SC nor JC is an acceptable principle for *de se* updating, then what is? I do not have a general principle for updating *de se* credences yet. In this section, however, I will suggest a new principle for updating the degrees of some special *de se* beliefs.

Before presenting this new updating principle, I need new notions related to beliefs, contents, credences, and time:

First, there are *de se* beliefs whose truth-values are different depending upon when they are had but not upon who has them. For instance, think about Jane and Jack's common belief that it rains today in Boston. It can have different truth-values at different times: It is possible that this belief is true on Monday but false on Tuesday. However, it cannot have different truth-values for different people at any given time: It is impossible that this belief is true for Jane but false for Jack on any given day. Let's call a *de se* belief of this type an "irreducibly *de nunc* belief" and the content of an irreducibly *de nunc*

---

[19] Admittedly, this result is dependent upon the suppositions that (i) $C_{prev}(P)$=0.5 and (ii) $C_{now}(P)$>0. However, neither supposition includes anything that possibly justifies the reasoning of (7). For (i) merely means that Jake is neutral between *P* and not-*P* and (ii) just means that he doesn't rule out *P*.

belief an "irreducibly tensed proposition." I consider a belief of this type to be partially about what time it is.

Second, we will call a *de dicto* or irreducibly *de nunc* belief a "*de nunc* belief," and we will call a genuine or irreducibly tensed proposition a "tensed proposition."[20]

Third, we will call the degree of an irreducibly *de nunc* belief an "irreducibly *de nunc* credence" and that of a *de nunc* belief a "*de nunc* credence."

Fourth, I introduce this definition: For any tensed proposition X and (temporal) interval v,

> The truth-value of X is *invariant* within v iff for any moments t and t' in v, X's truth at t logically implies X's truth at t'.

For instance, consider tensed proposition R expressed by "it rains in Boston at some time today." The truth-value of R is invariant within Monday; for, if R is true/false at some moment on Monday, then R is true/false at any other moment on Monday. Similarly, the truth-value of R is invariant within Tuesday. However, R can have different truth-values on Monday and on Tuesday.

In addition to these notions, we need an adequate formal language to formulate a new updating principle. Hence, I construct such a language L:

First, we need the traditional probability language's logical connectives, arithmetic operators, and probability function letters in our new language.

[20] For the deductive theories of tensed propositions, see Prior (2003) or Rescher and Urquhart (1971).

Second, as we need propositional letters in the traditional probability theory, we need tensed propositional letters in our new language. For this, we reserve "*E*," "*F*," "*X*," "*Y*," and "*Z*."

Third, we use "*t*" as a moment letter, which denotes a moment or a point-sized temporal location, and "*v*" as an interval letter, which denotes an interval or a continuous class of moments. Together, we call them time letters.

Fourth, given a time letter, we use the corresponding capital letter as a special tensed proposition letter, denoting the tensed proposition that the present moment is or belongs to the time denoted by the variable. For example, if "*t*" is the letter denoting the moment of 9:00 AM on Sep. 4th in 2006, then "*T*" is the letter denoting the tensed proposition that the present time is exactly that moment. Similarly, if "*v*" is the interval letter denoting the day of Sep. 4th in 2006, then "*V*" is the letter denoting the tensed proposition that it is that day.

Fifth, we introduce binary operators "at" and "in." Let "φ," "τ," and "ν" be schematic letters replaced with a tensed propositional letter, a moment letter, and an interval letter, respectively. Here is a meaning schema for "at":

"φ at τ" means that φ is true at τ.

For instance, let "*R*" mean the tensed proposition that it rains now in Boston and let "*t*" denote the moment of 9:00 AM on July 18th 2006. Then, "*R* at *t*" denotes the proposition that it rains in Boston at that moment. In addition, we introduce this abbreviation:

"φ in v" abbreviates "$(\forall t \in v)(\varphi$ at $t)$."

In other words, "φ in v" means that φ is true throughout v. For example, let v be July 18th 2006. Then, "R in v" means that it rains in Boston throughout July 18th 2006.

Now, we are ready to discuss a principle for updating *de nunc* credences. First, we consider an agent who has a tensed proposition E as certain total evidence and fully believes that she is located in v. For such an agent, I recommend this method of updating her credence in a tensed proposition, which I call "Shifted Strict Conditionalization":

(SSC) $C_{n+1}(X)=C_n(X \text{ in } v/E \text{ in } v)$ if E is the agent's certain total evidence at t_{n+1} and she fully believes at t_{n+1} that it is v,

where $C_t(E \text{ in } v)>0$, and the truth-values of X and E are invariant within v. In other words, if E is a rational agent's certain total evidence at t+1, and she fully believes at t_{n+1} that it is v, her credence at t_{n+1} in a tensed proposition X is the same as her previous conditional credence in [X's truth in v] given [E's truth in v], as long as the conditional credence is defined and neither X nor E can have different truth-values at any two moments in v.

In order to see how SSC works, consider this example: **Example 4**. Let P be that some form of precipitation occurs today in Boston and R be that it rains today in Boston. Suppose that on Sunday, Jane's conditional credence in [R's truth on Monday] given [P's truth on Monday] is 0.3.[21] On Monday, she learns and becomes certain that some form of precipitation occurs today in Boston, and today is Monday. What, then, is Jane's rational

[21] In this paper, I use brackets simply to avoid scope confusion.

credence on Monday in its raining today in Boston? My answer: $C_{MON}(R)=C_{SUN}(R$ is true on Monday/P is true on Monday)=0.3 by SSC.

Second, consider an agent B who possibly lacks certain total evidence or is uncertain of what time it is. In order to capture such uncertainties, we assume two partitions $\{E_i\}_{i\in I}$ and $\{v_j\}_{j\in J,}$ such that E_is are the tensed propositions, such that B's credences in E_is are directly set by her experience at $t_{n+1,}$ and such that v_js are temporal intervals covering the minimal interval that B fully believes at t_{n+1} that she is located in. Given these partitions, we will call E_is "(B's) observation propositions at t_{n+1}" and V_js "(B's) temporal location propositions at t_{n+1}." Roughly, an agent lacks certain total evidence iff she is uncertain of which of her observation propositions is true, and she is uncertain of what time it is iff she is uncertain of which of her temporal location propositions is true.

It's possible to unify these two dimensions of uncertainty into one. Consider $\{E_i\&V_j\}_{<i,j>\in I\times J}$, consisting of consistent conjunctions of the members of the above two partitions. Let's call any such conjunction "(B's) time-observation proposition at t_{n+1}." To B, one of her time-observation propositions at t_{n+1} is a candidate for the true conjunction of her observation and temporal location propositions at t_{n+1}. Roughly, an agent lacks certain total evidence and/or is not sure of what time it is iff she is uncertain of which of her time-observation propositions is true.

Naturally, we focus upon B's time-observation propositions whose truth she doesn't completely rule out at t_{n+1}. Thus, let $K\subseteq I\times J$ be the class of $<i,j>$s such that

$C_{n+1}(E_i \& V_j)>0$, where C_{n+1} is B's credence function at t_{n+1}. If $C_n(E_i \text{ at } v_j)>0$ for any $<i,j>$ in K, I call $\{E_i \& V_j\}_{<i,j>\in K}$ "(B's) time-observation partition at t_{n+1}."

Finally, we formulate a method of updating credences in tensed propositions, which I call "Shifted Jeffrey Conditionalization":

$$\text{(SJC)} \quad C_{n+1}(X) = \sum_{<i,j>\in K} C_n(X \textit{ in } v_j / E_i \textit{ in } v_j) C_{n+1}(E_i \& V_j) \text{ if } \{E_i \& V_j\}_{<i,j>\in K}$$

is the agent's time-observation partition at t_{n+1},

where, for any $i\in I$ and $j\in J$, the truth-values of X and E_i are invariant within v_j. In other words, if an agent lacks certain total evidence and/or is uncertain of what time it is at t_{n+1}, then her rational credence at t_{n+1} in a tensed proposition X is the weighted average of the results of applying SSC to X with various time-observation propositions at t_{n+1}, in which the weights are her credences at t_{n+1} in the time-observation propositions, as long as neither X nor any of E_is logically can have different truth-values at any two moments within each interval v_j.

To see how SJC works, think about the following examples: **Example 5**. Let "*P*" and "*R*" have the same meanings as in **Example 4**. On Sunday, Jane's conditional credence in [*R*'s truth on Monday] given [*P*'s truth on Monday] is 0.5, and her conditional credence in [*R*'s truth on Monday] given [not-*P*'s truth on Monday] is, of course, 0. On Monday, some perfectly reliable person tells her that it is Monday and something about today's weather, but she doesn't hear the latter information clearly. Has he said that some form of precipitation occurs, or that it doesn't occur, today in Boston? She is unsure. Her credence in the former is 0.7 and that in the latter is 0.3. In this case, what is Jane's credence on Monday in its raining in Boston on that day? My answer: $C_{MON}(R)$=C_{SUN}(R is true on Monday/P is true on Monday)$C_{MON}(P)$+C_{SUN}(R is true on Monday/not-P is true on Monday)C_{MON}(not-P)=0.35 by SJC.

Example 6. Again, keep the meanings of "P" and "R" the same. On Sunday, Jane is put to sleep with one of two drugs. The first drug's effect lasts for just one night, making her wake up on Monday. The second drug's effect lasts longer, making her wake up on Tuesday. Her conditional credence on Sunday in [R's truth on Monday] given [P's truth on Monday] is 0.8, and her conditional credence in [R's truth on Tuesday] given [P's truth on Tuesday] is 0.2. On Monday, Jane is told by a perfectly reliable person that some form of precipitation occurs today in Boston, and, not knowing which drug she took, Jane assigns the credence of 0.4 to its being Monday and that of 0.6 to its being Tuesday. In this case, what is Jane's rational credence on Monday in R? My answer: $C_{MON}(R)$=C_{SUN}(R is true on Monday/P is true on Monday)C_{MON}(it is Monday)+C_{SUN}(R is true on Tuesday/P is true on Tuesday)C_{MON}(it is Tuesday)=0.44 by SJC.

Despite the complicated appearance, SSC and SJC are actually quite intuitive. Certainly given E as total evidence and v as her temporal location, an agent should assign to X the previous conditional credence in X's truth in v given E's truth in v, not that in X given E. For the latter conditional credence is the (previous) conditional credence in X's *previous* truth given E's *previous* truth, which seems irrelevant to the (present) credence in X's *present* truth. Once accepting this, it seems natural that if the agent lacks certain total evidence and/or does not know what time it is, the new credence in X should be the weighted average of the previous conditional credence in [X's truth in v_j] given [E_i's truth in v_j], where E_is and v_js are the candidates for the presently true observation proposition and the temporal interval in which she is presently located.

In this section, I have presented a new principle for updating credences and illustrated how that principle works with examples. The intuitive answers yielded by SJC provide evidence in support of it, but additional argument is needed in order for us to accept its validity.

E. Shifted Rigidity as a Conditional Expert Principle

In this section, first, I will present a principle entailing SJC (which has SSC as a special case), and, second, I will discuss how that principle can be promoted by an intuitive expansion of Gaifman's Expert Principle (Gaifman 1988).

First, the principle: I call it "Shifted Rigidity." Let $\{E_i\&V_j\}_{<i,j>\in K}$ be the agent's time-observation partition at t_{n+1}. Then, for any tensed proposition X and any $<i, j>$ in K,

(SR) $C_{n+1}(X/E_i\&V_j)=C_n(X \text{ in } v_j/E_i \text{ in } v_j)$,

where the truth-values of E_i and X are invariant within each v_j. From the agent's point of view, this says, "The present relevance of E_i to X, on the additional condition that I am located in interval v_j, is the same as the previous relevance of [E_i's truth in v_j] to [X's truth in v_j]."

Obviously, SR entails SJC.[22] This means that we can argue for the latter by defending the former. But how can we defend SR? I believe that Gaifman's discussion of

[22] Let X be any tensed proposition and $\{E_i\&V_j\}_{<i,j>\in K\subseteq I\times J}$ be the agent's time-observation partition at t_{n+1}. Suppose that $C_{n+1}(X/E_i\&V_j)=C_n(X \text{ in } v_j/E_i \text{ in } v_j)$ for any $<i, j>\in K$, where the truth-values of X and E_i are

Expert Principles provides a clue to this question. Here is the schema of his so-called Expert Principles:

(Expert) $C(X/pr(X)=r)=r$, for all r such that $C(pr(X)=r)>0$.

Here, C is an agent's credence function and pr is the agent's "expert probability function." What is an expert probability function? Roughly speaking, it is a probability function that, once it is known to the agent, will be adopted by her as her own credence function. For instance, if you consider a local weather forecaster to be an expert for your local weather, then $C(\text{rain}/pr(\text{rain})=r)=r$ where C is your credence function and pr is the weather forecaster's. Here, pr does not have to be a *subjective* probability function. When pr is the objective chance function P, you get:

(Principal Principle) $C_0(X/P(X)=r)=r$ for any r such that $C_0(P(X)=r)>0$,[23]

where C_0 is an agent's initial credence function. When pr is the agent's future credence function at the next epistemic moment, you get:

(Reflection) $C_n(X/C_{n+1}(X)=r)=r$ for any r such that $C_n(C_{n+1}(X)=r)>0$.[24]

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invariant within each v_j for any $i\in I$ and $j\in J$. Since the partition exhausts $E_i\&V_j$ such that $C_{n+1}(E_i\&V_j)>0$, $C_{n+1}(\vee_{<i,j>\in K}(E_i\&V_j))=1$. Thus, $C_{n+1}(X)=C_{n+1}(X\&\vee_{<i,j>\in K}(E_i\&V_j))=\sum_{<i,j>\in K}C_{n+1}(X/E_i\&V_j)C_{n+1}(E_i\&V_j)$=(by supposition) $\sum_{<i,j>\in K}C_n(X \text{ in } v_j/E_i \text{ in } v_j)C_{n+1}(E_i\&V_j)$. Done. (The other direction is also provable under a few plausible assumptions but nothing in this paper hangs upon that direction of the equivalence. Still, the equivalence is interesting because it is analogous with the equivalence between Rigidity and JC.)

[23] The original version of PP is more general in that the condition can include additional information as long as it's *admissible* (Lewis 1987).

Why will an agent's future credence function be her expert function? If the given agent is not forgetful, she usually will be *more* knowledgeable in the future than in the present. (This is a good way to see why the Reflection Principle fails in the counterexample of Talbott (1991), in which the agent is forgetful.) By contrast, since an agent in the past is typically *less* knowledgeable than the same agent in the present, an agent's past credence function can't be her present expert function.

So far, so good. Now, consider this example.[25] **Example 7**. An investor consults a very trustworthy stock market expert. The problem is that the investor cannot reveal to the expert some insider information she has that a company will release a new product next month. The expert's opinion is generally unfavorable to that company, but his opinion conditioned upon the insider information is quite favorable. In that case, it will be rational for the investor to make a judgment on the basis of the expert's conditional (on the insider information) opinion. In this situation, it seems to be rational for the investor to adopt the expert's conditional credence function on the product release information as her own credence function. This intuition can be generalized as follows:

(Conditional Expert) $C(X/E\&pr(X/E)=r)=r$, for all r such that

$C(E\&pr(X/E)=r)>0$.

[24] The original version of Reflection is more general in that the agent's future credence function can be from a farther future than t_{n+1} (van Fraassen 1984).
[25] Hall discusses the same idea (Hall 2004, p. 100).

Here, C is an agent's credence function and pr is the agent's conditional expert probability function on E. By "conditional expert probability function on E," I mean a probability function pr such that, once the function $pr(\text{-}/E)$ and the truth of E are known to the agent, the agent will adopt it as her credence function. Given this definition, the Conditional Expert Principle is a natural expansion of the Expert Principle.

The Conditional Expert Principle provides a new way of understanding Rigidity. I suggest a sub-principle of Conditional Expert below:

(Backward Reflection) $C_{n+1}(X/E_i\&C_n(X/E_i)=r)=r$, for all r such that $C_{n+1}(E_i\&C_n(X/E_i)=r)>0$, where E_i is a member of the agent's observation partition at t_{n+1}.

To the agent at t_{n+1}, C_t is her previous credence function at t_n. I suggest that it also must be the agent's conditional expert probability function at t_{n+1} on E_i. Why? An agent usually has no choice but to depend on her previous credence to form the present one; however, she is also aware that her previous credence distribution was built without her present experience. Thus, an agent's relation to her past credence distribution is similar to the investor's relation to the stock market expert in **Example 7**: Due to the informational impoverishment of the agent's previous self, it may be irrational that her present credence in X is r given that her previous credence in X was r. Still, it is rational that her present credence in X is r given that E_i is the true member of her present observation partition, and her previous credence in X given E_i was r. This is because if E_i is true, then her previous credence function conditional on E_i was a judgment made on the basis of all information that she previously had *plus* the member of her observation partition actually confirmed by her present experience.

Therefore, let's make a plausible conjecture that Backward Reflection is usually true of a rational agent. If we assume that the given agent correctly knows her credence function and later remembers it with perfect confidence, Rigidity will obviously follow from Backward Reflection.[26] This provides a new way of understanding why we should accept Rigidity.

The last step in our expansion of Gaifman's principle is to apply the idea to tensed propositions, especially concerning Backward Reflection. However, I suggest that we need substantial modification to do so. Why? Consider this example. **Example 8**. Again, let *R* be that it rains today in Boston and let *P* be that some form of precipitation occurs today in Boston. In this example, an agent *B* at 9 AM on Monday (hereafter: t_m), knowing that it's Monday, regards herself at 9 PM on Sunday (hereafter: t_s) as an expert about local weather in Boston except that she didn't know whether there would be precipitation on Monday. At t_m, *B* learns that (i) *P* is true. At t_s, (ii′) *B*'s credence in *R* given *P* was 0.1, but (iii) her credence in [*R*'s truth on Monday] given [*P*'s truth on Monday] was 0.3. I make two claims: First, it is not the case that *B*'s rational credence at t_m in *R* is 0.1 given (i) and (ii′), but, second, her rational credence at t_m in *R* is 0.3 given (i) and (iii).

[26] Let *r* be $C_n(X/E)$. If the agent remembers her past credence distribution with perfect confidence, $C_{n+1}(C_n(X/E)=r)=1$. Thus, $C_{n+1}(X/E)=C_{n+1}(X/E\&C_n(X/E)=r)=$(by Backward Reflection)$r=C_n(X/E)$. Done.

To understand why, look at Figure 5:

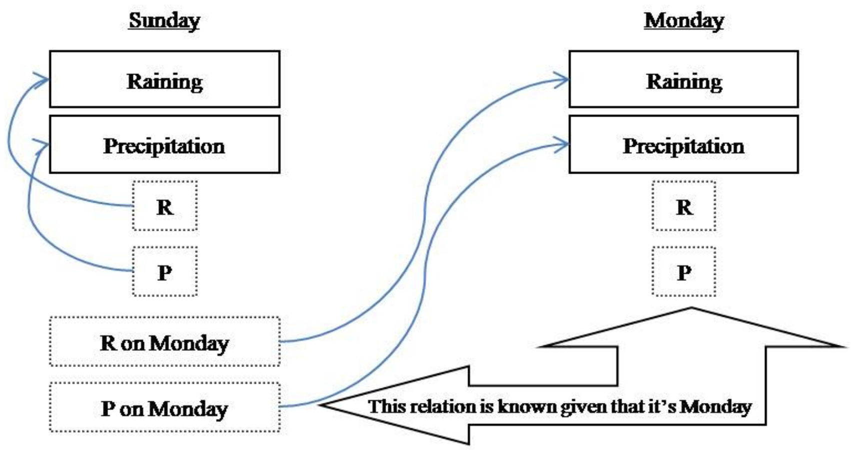

Figure 5: <u>Rain and Precipitation</u>. The tensed proposition at the arrow tail is true on the day of the tail's column iff the event at the arrow head occurs on the day of the head's column.

On the one hand, *B*'s opinion, represented by the conditional credence at t_s in *R* given *P* seems to be relevant only to whether raining happens on Sunday given that a form of precipitation occurs on Sunday. Hence, it is irrelevant to *whether raining happens on Monday given that a form of precipitation occurs on Monday*, which *B*'s credence at t_m in *R* is factually about. This suggests that *B*'s rational credence at t_m in *R* is not necessarily 0.1 given (i) and (ii′). On the other hand, *B*'s opinion at t_s, represented by the conditional credence at t_s in [*R*'s truth on Monday] given [*P*'s truth on Monday], is factually all about the italicized matter. This suggests that her rational credence at t_m in *R* is 0.3 given (i) and (iii).

What if *B* doesn't know at t_m that it is Monday? I think the natural expansion of the above discussion is to add the condition that it is Monday. Hence, on the condition that (i) a form of precipitation occurs now, (ii) it is Monday, and (iii) *B*'s past credence function at t_s was such that $C_{t_s}(R$ on Monday $/P$ on Monday$)=0.3$, the rational credence in raining is 0.3 according to the idea of the Conditional Expert Principle. Formally, $C_{t_m}(R/P\&V_m\&C_{t_s}(R \text{ in } v_m/P \text{ in } v_m)= 0.3)=0.3$, where v_m is Monday, and so V_m is that it is Monday.

We can extract a general idea from this example. On the condition that (i) observation proposition E_i is presently true, (ii) it is v_j, and (iii) the previous credence was such that $C_n(X \text{ in } v_j/E_i \text{ in } v_j)=r$ where the truth-values of X and E_i are invariant within each v_j, the credence of X must be r. More formally,

> (Shifted Backward Reflection) $\mathrm{C}_{n+1}(X/E_i\&V_j\&C_n(X \text{ in } v_j/E_i \text{ in } v_j)=r)=r$ for all x such that $\mathrm{C}_{n+1}(E_i\&V_j\&C_n(X \text{ in } v_j/E_i \text{ in } v_j)=r)>0$, where $E_i\&V_j$ is a member of the time-observation partition at t_{n+1}.

Of course, Shifted Rigidity follows from Shifted Backward Reflection if the agent remembers her past credence distributions with perfect confidence.[27]

Hence, the idea of SR is best understood when we stipulate that from an agent's present point of view, the agent herself at the previous moment is an expert only lacking the information confirmed by her present experience. For if that stipulation is true, it will be rational for the agent to coordinate her present credence distribution with her previous

[27] Let r be $C_n(X \text{ in } v_j/E_i \text{ in } v_j)$. If the agent remembers her past credence function with perfect confidence, $C_{n+1}(C_n(X \text{ in } v_j/E_i \text{ in } v_j)=r)=1$. Then, $C_{n+1}(X/E_i\&V_j)=C_{n+1}(X/E_i\&V_j\&C_n(X \text{ in } v_j/E_i \text{ in } v_j)=r)=$(by Shifted Backward Reflection) $r=C_n(X \text{ in } v_j/E_i \text{ in } v_j)$. Done.

one according to SR. Since the stipulation seems to be true and SR entails SJC, we have an argument for SJC.

F. Sleeping Beauty and Shifted Jeffrey Conditionalization

What answer does SJC give to the SB problem? In this section, I make three claims: First, it is disprovable under SJC that SB's credence in H stays the same from Sunday to Monday. Second, it is provable under SJC that her credences in H_1 and T_1 are the same on Monday. Third, it is not obviously provable or dis-provable that her credences in T_1 and T_2 are the same on Monday.

In my discussion in this section, the target tensed propositions will be H, T, H_1, T_1, and T_2, the evidence will be W, and the partition of intervals will be {Monday, Tuesday}. The truth-values of these target tensed propositions and evidence are invariant within Monday and within Tuesday. Hence, we can apply SJC to SB's updating from Sunday to Monday with the time-observation partition $\{W\&MON, W\&TUE\}$.[28]

Equipped with SJC, I criticize the Halfers' thesis (1), which asserts that with no relevant new evidence, no one can rationally change her credence in a genuine proposition. I show that the SB problem is a counterexample of this thesis. Look at this instance of SJC:

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[28] Why do we use SJC with exactly this fine-grained time-observation partition? On the one hand, the truth-value of W is not invariant within Monday+Tuesday (0:00 AM on Monday to 11:59 PM on Tuesday). Hence, you cannot use SJC with the time-observation partition $\{W\&(MON\vee TUE)\}$. On the other hand, the truth-value of W is invariant within each of {Monday AM, Monday PM, Tuesday AM, Tuesday PM}. Hence, you can use SJC with the time-observation partition $\{W\&MON_{AM}, W\&MON_{PM}, W\&TUE_{AM}, W\&TUE_{PM}\}$, but it will generate the same result as using SJC with $\{W\&MON, W\&TUE\}$. In sum, it violates SJC's proviso to use a more coarse-grained time-observation partition, and it is a waste of calculating effort to use a more fine-grained time-observation partition.

(8) $C_m(H) = C_s(H \text{ on Monday}/W \text{ on Monday})*C_m(W\&MON)+$

$C_s(H \text{ on Tuesday}/W \text{ on Tuesday})*C_m(W\&TUE)$.[29]

On the one hand, the first conditional credence is 1/2, which is equal to her credence on Sunday in *H*. For "*W* on Monday" in the first conditional credence phrase is redundant because she fully expected on Sunday night that she would wake up on Monday; furthermore, "on Monday" in the resulting unconditional credence phrase also would be redundant because *H* is a genuine proposition whose truth-value is insensitive to time. On the other hand, the second conditional credence is 0. For waking up on Tuesday means that the coin lands tails. In sum, her credence on Monday in *H* is the weighted average of 1/2 and 0, where the weights are her credences on Monday in *W&MON* and in *W&TUE*. Since SB cannot rationally rule out either possibility, $0<C_m(H)<1/2$. Because *W* doesn't seem to be new evidence relevant to *H*, this is a counterexample of (1).

In general, this explains how (1) can be violated by an SJC-er: Even when an agent's total evidence *E* is not new and relevant to *X*, she may change her credence in *X*. For if she is uncertain whether her temporal location is in v_1 or in v_2, she is uncertain between two time-observation propositions, $E\&V_1$ and $E\&V_2$. Even if the certain evidence, *E*, is not new in that she fully expected that *E* would be true, one or both of $E\&V_1$ and $E\&V_2$ can be new in that she didn't fully expect *E*'s truth in v_1 and/or in v_2. In such a case, an SJC-er can change her credence in *X* because arithmetically, it's the newness/oldness of the *time-observation* propositions, not of the evidence, that decides

[29] In this section, I will use "… on Monday" as the abbreviation of "… is true (at every moment) on Monday"; likewise for "… on Tuesday."

whether it is rational for the agent to change her credence in *X*. (This point generalizes to any *n*-case.)

Second, I defend the Thirders' thesis (2): When SB wakes up on Monday, her credence in H_1 is the same as that in T_1. As I discussed in Section A, Thirders have defended this thesis with the intuition that SB's credences in *H* and *T* would be equally 1/2 if she received additional information that it is Monday; hence, the actual conditional credences in *H* and *T*, given *MON*, also should be 1/2. Arithmetically, this leads to thesis (2): $C_m(H_1)=C_m(T_1)$. I consider this to be a sound argument.

However, it would be more convincing if they could derive SB's credence distribution on Monday from that on Sunday, rather than from her credence distribution in a counterfactual situation. With SJC, that derivation is possible. Here are its two instances for SB's credences at *m* in H_1 and *T*:

$$C_m(H_1) = C_s(H_1 \text{ on Monday}/W \text{ on Monday})*C_m(W\&MON)+ C_s(H_1 \text{ on Tuesday}/W \text{ on Tuesday})*C_m(W\&TUE). \tag{9}$$

$$C_m(T_1) = C_s(T_1 \text{ on Monday}/W \text{ on Monday})*C_m(W\&MON)+ C_s(T_1 \text{ on Tuesday}/W \text{ on Tuesday})*C_m(W\&TUE). \tag{10}$$

According to (9), the credence on Monday in H_1 is the weighted average of the conditional credences on Sunday in [H_1's truth on Monday] given [*W*'s truth on Monday] and in [H_1's truth on Tuesday] given [*W*'s truth on Tuesday]. The first conditional credence is 1/2. For [H_1's truth on Monday] is equivalent to *H*'s truth simpliciter, and SB fully expected *W* to be true on Monday; hence, it equals the credence on Sunday in *H*, 1/2.

The second conditional credence is 0. For H_1 cannot be true on Tuesday. Hence, SB's conditional credence on Monday in H_1 is the average of 1/2 and 0, the former weighted by the credence in *W&MON* and the latter by that in *W&TUE*. As a result, $C_m(H_1)=1/2$ $C_m(W\&MON)$. Similarly, it follows from (10) that $C_m(T_1)=1/2C_m(W\&MON)$. Therefore, $C_m(H_1)=C_m(T_1)$.

We already established the truth of the Lesser view. However, thesis (2) provides an alternative proof: Remember that H_1, T_1, and T_2 exclusively exhaust all possibilities open to SB at the moment of wakeup on Monday. She cannot rule out T_2 and so $C_m(H_1)+C_m(T_1)<1$. Since $C_m(H_1)=C_m(T_1)$, $C_m(H_1)<1/2$. Because H_1 is the only possibility in which the coin lands heads, $C_m(H)<1/2$.

Given this, what is the *precise* value of SB's credence in H when she wakes up? As discussed in Section A, the answer will be 1/3 if thesis (3) is true: When SB wakes up on Monday, her credence in T_1 is the same as that in T_2. Unfortunately, there is no obvious way to prove or disprove this thesis by SJC. (Try it.)

Elga says that we can prove (3) by a principle of indifference:

> …even a highly restricted principle of indifference yields that you ought then to have equal credence in each. (Elga 2000, 144.)

No doubt, "each" refers to each of T_1 and T_2. Thus, Elga is arguing here that (3) follows from a highly restricted principle of indifference. In response to this, I ask two questions: First, does (3) really follow from his principle of indifference? Second, is his principle of indifference possibly true?

Both questions are hard to answer because Elga did not provide an explicit formulation of his principle of indifference in his paper (2000). Fortunately, he provided a formulation of that principle in a more recent paper (Elga 2004, 387): Define a centered world to be a maximally consistent centered proposition.

(INDIFFERENCE) For any centered worlds *X* and *Y*, a rational agent ought to assign the same credence to *X* and *Y* if (i) they are associated with the same possible world (i.e. for some possible world *W*, *X* and *Y* both imply that *W* is the actual world) and (ii) they represent epistemic situations that are subjectively indistinguishable (i.e. whichever of *X* and *Y* is true of you, your experience will be exactly the same).

At first glance, this principle appears to entail (3): Assume that SB fully knows everything about her world except whether the coin lands heads or tails. Under this assumption, we can think as if T_1 and T_2 are centered worlds satisfying (i) and (ii). Hence, it follows that $C_m(T_1)=C_m(T_2)$. However, Weatherson (2005) criticizes this approach: First, he says, we cannot infer (3) from INDIFFERENCE without the above assumption. For define *S* to be the set of possible worlds such that for each *W* in *S*, the actuality of *W* is compatible with T_1 and T_2, and consider the case where *S* is *uncountably* large. In this case, it is perfectly coherent for SB to assign the same credence to any two centered worlds associated with the same possible world in *S* but different credences to T_1 and T_2. (For more details, see Weatherson (2005, pp. 615-616).)

Second, INDIFFERENCE is incompatible with Countable Additivity. For assume a possible world *W* containing an *infinite but countable* number of agents who are in the epistemic situations that are subjectively indistinguishable; if Countable Additivity is true, it is incoherent to assign the same credence to the centered worlds representing these agents' epistemic situations in *W*. (For more details, see Weatherson (2005, pp. 619-621).) In sum, INDIFFERENCE entails (3), but it does so only under a highly unlikely assumption, and INDIFFERENCE is incompatible with the widely accepted axiom of Countable Additivity. Hence, the validity and factual correctness of Elga's argument are both questionable. That said, I leave it as an open question whether there exists a consistent principle of indifference entailing (3).[30]

So far, I have disproved (1) and proved (2) by SJC. I have pointed out that (3) is not obviously provable or disprovable under SJC. Consequently, I reject the Halfer view and accept the Lesser view. However, I leave it as an open question whether the exact value of SB's credence in *H* is fixed by a principle of indifference or any other consideration.

G. Conclusion

In this chapter, I have defended the Lesser view of the SB problem by SJC, a new principle for updating *de nunc* credences. My discussion not only defends the Lesser view, but it also provides a clue for what has gone wrong with the Halfer view. Read Elga's following comment on SB's increasing credence in *H*:

[30] However, I am skeptical. First, Elga points out that INDIFFERENCE leads to the weird result of "a brain race," although he bites the bullet (Elga 2004, p. 394). Second, Weatherson argues (convincingly in my opinion) that INDIFFERENCE is not motivated because the intuition behind it is better captured in the framework of imprecise credences (Weatherson 2005, p. 624). In my opinion, any principle similar to INDIFFERENCE is likely to share these problems.

This belief change is unusual. It is not the result of your receiving new information. … So what justifies it? The answer is that you have gone from a situation in which you count your own temporal location as *irrelevant* to the truth of *H*, to one in which you count your own temporal location as *relevant* to the truth of *H*. (Elga 2000, 146)

SJC provides a good explanation as to why Elga's statement is correct: It follows from SJC that $C_m(H)=C_s(H/W \text{ on Monday})C_m(W\&MON)+C_s(H/W \text{ on Tuesday})C_m(W\&TUE)=1/2C_m(MON)+0C_m(TUE)\in(0,1/2)$. This decrease was possible because, given that SB is awake, *TUE* confirms [*W*'s truth on Tuesday], which is negatively relevant to the truth of *H*. Since SB's earlier knowledge that it was Sunday was not relevant to *H* in this way, she has gone from [a situation in which she counts her temporal location as irrelevant] to [one in which she counts it as relevant]. In this case, even if the evidence is old and irrelevant to *X*, it is not sufficient for the rationality of not changing the credence in *X*.

Since the debate between Halfers and Thirders has been due not to the lack of supporting arguments but to the failure of each side to point out the other side's problem, this is a good achievement. However, the even greater accomplishment seems to be the updating principle itself. David Lewis wrote that the rule for updating *de se* credences is formally identical to the rule for updating *de dicto* credences.[31] This is wrong. As I have demonstrated in this chapter, SJC is a good candidate for a rational principle for updating the narrower category of *de nunc* credences. Obviously, SJC needs further

[31] Lewis (1979) writes: "Then it is interesting to ask what happens to decision theory if we take all attitudes as *de se*. Answer: very little. We replace the space of worlds by the space of centered worlds … All else is just as before." (Lewis 1979, pp. 533-4).

generalizations, but at least it initiates a good starting point from which we can find the universal rule for updating *de se* credences.

CHAPTER III

UPDATING WITH A SEQUENCE OF OBSERVATIONS

A. Introduction

In Chapter II, I argued that a rational agent updates her credence in a tensed proposition by SJC. In formulating SJC, I introduced this proviso: In updating from a past credence function C_n at t_n to the present credence function C_{n+1} at t_{n+1}, the agent observes nothing between t_n and t_{n+1}.

Due to this proviso, SJC is not versatile. Compare it with SC, the traditional rule for *de dicto* updating. According to the *de dicto* version of SC, if an agent observes E^1, E^2, E^3, …, E^m during $(t_n, t_{n+m}]$, she can simply conditionalize upon the conjunction $E^1\&E^2\&E^3\&\ldots\&E^m$ to update her credence in any proposition X (where X, E^1, E^2, E^3, …, E^m are genuine propositions). This fact suggests that as a result of observing E^1, E^2, E^3, …, E^m, she learns $E^1\&E^2\&E^3\&\ldots\&E^m$.

Unfortunately, the same cannot be said for *de se* observations. Suppose that Jane watches a cloudy sky first and a clear sky later, from 4PM to 5PM. What does she learn as a result? If it is the conjunction of watching a cloudy sky and watching a clear sky, then she must have learned the same thing as what she would have learned if she had watched a clear sky first and a cloudy sky later during that period. Of course, this is absurd. In general, when a given agent's observations are *de se*, their *order* is important in determining what the agent has learned as a result of those observations. Therefore, what she learns cannot be just the conjunction of those observations.

For this reason, I chose to apply SJC only under the proviso that the agent observes nothing during (t_n,t_{n+1}). When that proviso is satisfied, what she learns as the result of her observations during $(t_n,t_{n+1}]$ is simply what she observes at t_{n+1}. In that way, we did not have to worry about the order of observations.

However, this proviso is too restrictive. For example, think about this version of the SB problem: **SB problem 1**. On Sunday, SB knows that she will experience the following experiment. One minute later, she is put to sleep by a group of evil experimenters. Then, they toss a fair coin. Case 1: (*H*) The coin lands on heads. In this case, they awaken her only once on Monday. Case 2: (*T*) The coin lands on tails. In this case, they awaken her twice, the first time on Monday and the second time on Tuesday; between the two awakenings, they inject her with a drug that erases her memory of the first awakening. In either case, one minute after she wakes up on Monday, she is told that it is Monday. The experiment ends on Wednesday when she wakes up with the memory of the previous awakening.

Let *s* be SB's last conscious moment on Sunday, *m* be the moment of her wakeup on Monday, and *m*+ be that of her being told that it is Monday. During (*s*,*m*+), she observes *W* ("SB wakes up with the memory of Sunday as the last memory"). Since this violates the proviso, SJC does not apply to SB's credal transition from *s* to *m*+. One may say that this is not a big problem because we can apply SJC to her credal transition from *s* to *m* and then to her credal transition from *m* to *m*+. However, it is clear that it would be better if we had an updating rule free from this kind of restriction.

In this chapter, I will suggest that if an agent makes a sequence of *de nunc* observations E^1, E^2, E^3, …, E^{m} during $(t_n,t_{n+m}]$, she may update her *de nunc* credence in a tensed proposition X by using a new updating rule that I call "Sequential Shifted Jeffrey Conditionalization" (hereafter: SSJC). I will defend SSJC to some extent, but I will also point out that SSJC does not apply to every case. Hence, I will try to provide some criterion to distinguish the cases in which SSJC is a rational updating strategy from those in which it is not.

B. Review of SC and SJC

In this section, I will review [the *de se* version of SC] and SJC. In particular, I will explain how the latter solves a problem of the former.

Consider this case: **Example 1**. On Monday, Jane listens to the radio news, which reports that today is Monday and that if there is any form of precipitation in Boston on Tuesday, it will be rain. Consequently, Jane is sure to the degree of 0.9 that today is Monday, but she does not completely trust the news, and so she assigns the credence of 0.1 to the possibility that today is Tuesday. Conditional on its being Tuesday, she ascribes no authority to the news. Hence, (i) she believes to the degree of 0.5 that [it rains in Boston today] given that [Today is Tuesday and there is a form of precipitation in Boston today].[32] Conditional on its being Monday, Jane regards the weather news as authoritative. Since she is pretty sure that it's Monday, she ascribes some authority to the news, even unconditionally. Thus, (ii) she believes to the degree of 0.8 that [it rains in Boston on Tuesday] given that [there is a form of precipitation in Boston on Tuesday]. After listening to the news, she is put to sleep and stays in that state until next morning. Waking up on Tuesday, Jane is told by her guru that today is Tuesday and that there is a

[32] Let's suppose that she knows that if any form of precipitation in Boston on Tuesday, it will be a rain or snow, and she will be neutral between the two possibilities conditional on any form of precipitation there on Tuesday.

form of precipitation today. At that moment, how confident will Jane be that it rains today?

If Jane is a *de se* SC-er, she will be confident to the degree of 0.5 that it rains today. If Jane is an SJC-er, she will be confident to the degree of 0.8 that it rains today. Why is the former view wrong? To answer, let me formulate the *de se* version of SC again: For simplicity, suppose that the agent *B* makes no observation during (t_n, t_{n+1}). Let *X* be a centered proposition and *E* be the *de se* evidence that the agent *B* receives at $t_{\mathrm{n+1}}$. Then,

$$(1)\ C_{n+1}(X)=C_n(X/E)$$

where C_n and C_{n+1} are *B*'s credence functions at t_n and t_{n+1}. The goal from *B*'s point of view at t_{n+1} is to find the rational credence that *X* is true *now* given the evidence that is true *now*, where "now" refers to t_{n+1}. However, the right-hand side of (1) denotes the agent's earlier credence that *X* was true *then* given that *E* was true *then*, where "then" refers to t_n. In this sense, if *B* is a *de se* SC-er, she comes to set her present credence in *X* by consulting an *outdated* conditional credence. I will call this problem "the outdated conditional credence problem." Note: this was not a problem for the *de dicto* version of SC because the target proposition and evidence have fixed truth-values in the *de dicto* framework.

Consider **Example 1** again. For brevity, let *R* be the tensed proposition that it rains in Boston today, *TUE* be the tensed proposition that today is Tuesday, and *P* be the tensed proposition that there is a form of precipitation in Boston today. *TUE&P* is Jane's total evidence on Tuesday. If she is a *de se* SC-er, her credence on Tuesday in *R* will be her credence on Monday in *R* given *TUE&P*. From Jane's point of view on Tuesday, when she judges how likely it is that *R* is true *now,* she is consulting her credence in *R*'s *then* truth given *TUE&P*'s *then* truth, where "now" refers to Tuesday and "then" refers to Monday. Intuitively, this means that she comes to set her credence in *R* by consulting a temporally mismatching conditional credence.

Fortunately, SJC helps us overcome the outdated conditional credence problem. For simplicity, focus upon its sub-principle SSC: Suppose that the agent *B* receives no evidence during (t_n,t_{n+1}) and knows that the time is *v*. Let *X* be a tensed proposition and *E* be the *de nunc* evidence that the agent *B* receives at t_{n+1} (where the truth-values of *X* and *E* are invariant within *v*). Then,

(2) $C_{n+1}(X)=C_n(X \text{ in } v/E \text{ in } v)$

where C_n and C_{n+1} are *B*'s credence functions at t_n and t_{n+1}. In words, *B* sets her present credence in *X* to be her previous credence that [*X* is true in *v*] given that [*E* is true in *v*]. This is intuitively reasonable, because from *B*'s point of view at t_{n+1}, *X* is true iff "*X* is true in *v*" is true *at whatever time,* and *E* is true iff "*E* is true in *v*" is true *at whatever time*. (More about this point below.)

In **Example 1**, if Jane is an SSC-er, her credence on Tuesday in *R* will be 0.8. For her credence on Tuesday in *R* will be her credence on Monday in [*R*'s truth on Tuesday] given [*TUE&P*'s truth on Tuesday]. From her point of view on Tuesday,

(3) *R* is true now iff [*R*'s truth on Tuesday] holds *at whatever time*, and

(4) *TUE&P* is true now iff [*TUE&P*'s truth on Tuesday] holds *at whatever time*.

These facts have two consequences: First, given these equivalences, it seems okay on Tuesday for Jane to evaluate the credal impact of *TUE&P* on *R* by evaluating that of [*TUE&P*'s truth on Tuesday] on [*R*'s truth on Tuesday].[33] Second, there is no problem of temporal mismatch in setting her credence on Tuesday in *R* with evidence *TUE&P* by consulting her previous credence in [*R*'s truth on Tuesday] given [*TUE&P*'s truth on Tuesday]. For the bracketed propositions are genuine propositions, which have fixed truth-values. Therefore, this application of SSC is free from the outdated conditional credence problem.

To generalize the above discussion, I introduce the following definition: Let *X* be any tensed proposition and *v* be a temporal interval. Additionally, let *V* be the tensed proposition that it is *v* now. Then,

(5) The *de-indexicalization* of *X* under *V* is the genuine proposition that *X* is true throughout *v*.

Given (5), we can verbalize (2) into this claim:

[33] By "credal impact," I mean the quantity of the force of evidence or observation that increases or decreases the agent's credences. While this definition is not a good example of philosophical clarity, I am not alone in using this somewhat opaque notion. See Lewis (1980; 272) for an example.

(6) B's credence at t_{n+1} in X=B's credence at t_{n} in [the de-indexicalization of X under V] given [the de-indexicalization of E under V].

The point of de-indexicalization here is to convert the target tensed proposition X and *de nunc* evidence E into the equivalent genuine propositions X' and E' so that B can judge how probable X is given evidence E by checking her previous credence in X' given E'. In this process, de-indexicalization allows the agent to use her previous conditional credence to set her present credence without the outdated conditional credence problem.

As we have seen, SSC solves the outdated conditional credence problem by de-indexicalizing the target tensed proposition and evidence. We can regard SJC as a generalization of SSC where the agent is unsure about what observation she has made and what time it is. The next question is, "How can we generalize the idea of de-indexicalization for when the agent receives a sequence of evidence?"

C. Strategy

To answer the above question, I want to discuss SB's credal transition from s to m+ as an example. I begin by asking three questions: First, what does SB learn during (s,m+]? Second, is there a genuine proposition equivalent to what she learns as a result of her observations during that interval? Third, what is the correct way to update her credence in H from s to m+ by making use of what she learns?

In answer to the first question, SB observes W at m and observes MON at m+. As a result, she comes to fully believe at m+ that ($\mathcal{E}$) W was previously true and MON is presently true. In sum, $\mathcal{E}$ is what she learns as a result of her observations during (s,m+].

In answer to the second question, SB knows at $m+$ that it was previously Monday and that the truth-value of W is invariant within Monday. Thus, she will accept this bi-conditional at that moment:

(7) "W was previously true" is true now iff "W is true on Monday" is true at whatever time.

Because she also knows at $m+$ that it is presently Monday and the truth-value of *MON* is invariant within Monday, she will accept

(8) *MON* is true now iff "*MON* is true on Monday" is true at whatever time,

at $m+$. Now, let $\mathcal{D}$ be the genuine proposition that W is true on Monday and *MON* is true on Monday. From the point of view at $m+$, $\mathcal{D}$ and $\mathcal{E}$ are equivalent because their conjuncts are equivalent. Since $\mathcal{D}$ is a genuine proposition, there exists a genuine proposition that is equivalent at $m+$ to $\mathcal{E}$.

And now to the third question: what is the correct method of SB's updating her credence in H from s to $m+$? Since $\mathcal{E}$ is equivalent to $\mathcal{D}$, she can evaluate the evidential impact of $\mathcal{E}$ upon H by evaluating that of $\mathcal{D}$ on H. Hence,

(9) $C_{m+}(H)=C_s(H/\mathcal{D})=C_s(H/MON$ is true on Monday&W is true on Monday$)$.[34]

[34] Since SB is sure of $\mathcal{E}$ at $m+$, $C_{m+}(H)= C_{m+}(H/\mathcal{E})$. Thus, if $\mathcal{E}$'s impact at $m+$ on H can be measured by checking $\mathcal{D}$'s impact at s on H, then (10) is derived.

It is trivial that *MON* is true on Monday, and on Sunday SB fully expected to wake up on Monday. Hence,

(10) $C_{m+}(H)=C_s(H)=1/2.$

This result complies with the traditional Thirder view (Elga 2000).

Let us suppose that (9) is the correct way for SB's updating from s to $m+$. If we can generalize it, perhaps we can find a model for an agent's updating from t_n to t_{n+m} where the agent makes observations many times during $(t_n,t_{n+m}]$. The remaining question is "How?"

In the rest of this chapter, I will proceed in the following order: In Section D, I will present two new updating principles. In Section E, I will construct an argument for these new principles by utilizing the idea of the Conditional Expert Principle, as I did in Chapter II. In Section F, however, I will argue that those updating principles lead to mutually inconsistent results when applied to the SB problem. In Section G, I will provide a diagnosis for the problem. In Section H, I will formulate weaker versions of the two principles suggested in Section D.

D. Updating with a Sequence of Observations

In this section, I will discuss how to update *de nunc* credences after making a sequence of observations. First, I will expand our formal language L into a larger language L', and then I will introduce a few new definitions. Next, I will present two *de nunc* updating principles by using the new language and definitions. Finally, I will illustrate how these principles work in several examples.

I begin by expanding language *L*, which I constructed in the last chapter, into a new language *L′*:

First, every expression in *L* is also a legitimate expression in *L′*.

Second, *L′* includes an indexical “prev.” Remember the assumption that I made in the previous chapter (see footnote 14): The agent makes observations only at a countable number of times, say, ... t_{n-2}, t_{n-1}, t_n, t_{n+1}, t_{n+2}, t_{n+3}, ... Let us call those moments “epistemic moments.” From now on, I will use the numeric subscripts to indicate the order and contiguity of the epistemic moments. In other words, for any $n,m \in N$, t_n is an earlier epistemic moment than t_m iff $n<m$ and t_n is the last moment the agent observes anything before t_{n+1}. Then, here is the meaning schema for “prev”:

(11) At t_n, “prev” refers to t_{n-1}.

English has no precise counterpart to “prev” in *L′*, but I will often use “the previous moment” to mean the same thing.

Third, *L′* includes an indexical “pres.” Here is the meaning schema:

(12) At t_n, “pres” refers to t_n.

Hence, “pres” in *L′* amounts to “the present moment” in English.

Fourth, L' includes (somewhat artificial) indexicals "prev_k," where $k \geq 0$. Here is the meaning schema for "prev_k":

(13) At t_n, "prev_k" refers to t_{n-k}.

As a result, "prev_0" refers to the same epistemic moment as "pres" does, and "prev_1" refers to the same moment as "prev" does. This finishes our expansion of L into L'.

Next, I define two important notions. In order to do so, I ask the following questions: First, if an agent makes a sequence of observations, what does she come to learn at the end? Second, assuming an answer to the previous question, is there a genuine proposition equivalent to what she learns?

Focus on the first question. Let me precisify it first: Suppose that an agent B observes E^1, E^2, E^3, ... E^m, and nothing else during interval $(t_n, t_{n+m}]$. Then, what does B come to learn at t_{n+m} as the result? Consider this answer:

(14) $E^1 \& E^2 \& E^3 \& \ldots \& E^m$.

Unfortunately, this suggestion is inadequate when E^1, E^2, E^3,... E^m are irreducibly tensed propositions. For (14) ignores the temporal gaps among E^1, E^2, E^3,... E^m. To understand this point, think about the following example: **Example 2**. Let E^1 be the tensed proposition that the sky is cloudy now and E^2 be the tensed proposition that the sky is clear now. Suppose that Jane observes E^1 at 4:30 PM and E^2 at 5:00 PM during (4:00 PM, 5:00 PM], and nothing else during that period. Then, there is a serious problem if we regard E^1&E^2 as what Jane learns as the result. For, at whatever time it is evaluated, E^1&E^2 entails that the sky is clear and cloudy *at the same time*. Since Jane's perfectly normal experience cannot lead to such absurdity, E^1&E^2 is not what she comes to learn as the result of her observations during (4:00 PM, 5:00 PM].

Instead, I suggest that what an agent B learns as the result of her observations $E^1,E^2,E^3,\ldots, E^m$ during $(t_n,t_{n+m}]$ is

(15) $(E^1 \text{ at prev}_{m-1})\&(E^2 \text{ at prev}_{m-2})\&(E^3 \text{ at prev}_{m-3})\&\ldots\&(E^m \text{ at prev}_0)$

in L'. Why? Note that (15) is the translation to L' of the following expression in (my dialect of) English:

(16) E^1 was true m-1 epistemic moments ago&
E^2 was true m-2 epistemic moments ago&
...
E^m is true 0 epistemic moment ago (or now).

Remember that epistemic moments are the moments at which the given agent observes anything. If B has perfect memory as is usually assumed, then B will remember that she observed E^1 m-1 epistemic moments ago, E^2 m-2 epistemic moments ago, etc. In general, B will remember that she observed E^k m-k epistemic moments ago, for any k such that $1\leq k\leq m$. Thus, as a result of having observed E^1, E^2, E^3,... E^m, she comes to learn the tensed proposition expressed by (15) or (16). For example, Jane will believe at 5:00 PM that (E^1) "the sky is cloudy" was true when she observed anything last time, and (E^2) "the sky is clear" is true now. This belief is expressed in L' by "(E^1 at prev$_1$)&(E^2 at prev$_0$)" or "(E^1 at prev)&(E^2 at pres)." In my opinion, this is what Jane learns as the direct result of her observations during (4:00 PM, 5:00PM].

In general, I suggest the following definition:

(17) $\mathcal{E}$ is the agent B's *sequential total observation* during $(t_n,t_{n+m}]$ iff

(i) B observes E^1, E^2,... E^m and nothing else during $(t_n,t_{n+m}]$, and

(ii) $\mathcal{E}$=(E^1 at prev$_{m-1}$)&(E^2 at prev$_{m-2}$)&...&(E^m at prev$_0$).

(Be careful: While "E" carries the increasing subscripts from 1 to m, "prev" carries the decreasing subscripts from m-1 to 0.)

Now, let's focus on the second question. Again, I first precisify the given question: If (E^1 at prev$_{m-1}$)&(E^2 at prev$_{m-2}$)&...&(E^{m} at prev$_0$) is B's sequential total observation during $(t_n,t_{n+m}]$, is there a genuine proposition equivalent to that observation?[35]

To answer this question, first, I expand the notion of de-indexicalization defined in Section B: Let E be a tensed proposition and v be a temporal interval such that the truth-value of E is invariant within v. Thus, V is the tensed proposition that it is v now. Then,

[35] When it results in no confusion, I shall mix target language, L', with the meta-language, English. The same applies to the rest of this dissertation.

(18) The *de-indexicalization* of [E at prev$_k$] under [V at prev$_k$] is [E in v].

Here is the core idea: Under the hypothesis that it was v k epistemic moments ago, E was true k epistemic moments ago iff E was true in v.[36] Hence, under the temporal hypothesis [V at $prev_k$], the agent will judge that [E at $prev_k$] is true iff the genuine proposition [E in v] is true *at whatever time*.

Next, I expand the notion of de-indexicalization for the case where an agent makes a sequence of observations during an interval:

(19) The *sequential de-indexicalization* of

(i) $(E^1 \text{ at prev}_{m-1})\&(E^2 \text{ at prev}_{m-2})\&...\&(E^m \text{ at prev}_0)$

is

(ii) $(E^1 \text{ in } v^1)\&(E^2 \text{ in } v^2)\&...\&(E^m \text{ in } v^m)$

under the temporal hypothesis

(iii) $(V^1 \text{ at prev}_{m-1})\&(V^2 \text{ at prev}_{m-2})\&...\&(V^m \text{ at prev}_0)$

where the truth-value of E^k is invariant within v^{k} for any $k \in \{1,...,m\}$.

This looks complicated but the core idea is the same as before: Under the temporal hypothesis (iii), (i) is true now iff (ii) is true *at whatever time*. Given this conditional equivalence, the agent can evaluate the credal impact of (i) by evaluating that of (ii) when she knows that (iii) is true. For an agent's sequential total observation is always

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[36] Suppose [V at prev$_k$]. Clearly, this supposition entails that $prev_k \in v$. By the definition of "in," [E in v] implies [E at $prev_k$]. Since we are assuming that the truth-value of E is invariant within v, [E at $prev_k$] implies [E in v]. Done.

equivalent to its sequential de-indexicalization under the correct temporal hypothesis (about the relevant epistemic moments).

Having defined these notions, I am ready to formulate my first updating rule in this chapter, "Sequential Shifted Strict Conditionalization": Consider a sequence of observations E^1, E^2, ...E^m and a sequence of intervals v^1, v^2, ... v^m. Assume that the truth-value of X is invariant within v^{m} and that of E^k is invariant within v^k for each $k \in \{1,...,m\}$. Then, for any tensed proposition X,

(SSSC) $C_{n+m}(X)=C_n(X \text{ in } v^{\mathrm{m}}/(E^1 \text{ in } v^1)\&(E^2 \text{ in } v^2)\&...\&(E^m \text{ in } v^m))$

if B is sure at t_{n+m} that for each $k \in \{1,...,m\}$, [E^k was/is true and it was/is v^k] at the m-k epistemic moments earlier time,

where C_n and C_{n+m} are B's credence functions at t_n and t_{n+m}. Less formally, we can rewrite SSSC in this way: Let $\mathcal{E}$ be (E^1 at $\mathrm{prev}_{\mathrm{m-1}}$)&...&($E^{\mathrm{m}}$ at prev_0) and $\mathcal{V}$ be (V^1 at $\mathrm{prev}_{\mathrm{m-1}}$) &...&($V^{\mathrm{m}}$ at prev_0). Then,

(SSSC) $C_{n+m}(X)=C_n$(the de-indexicalization of X under V^{m}/the sequential de-indexicalization of $\mathcal{E}$ under $\mathcal{V}$), if B has certainly learned until t_{n+m} that $\mathcal{E}\&\mathcal{V}$ is true.

In my opinion, this is a reasonable rule for *de nunc* updating. It is free from the outdated conditional credence problem, as the target tensed proposition and sequential total observation are converted to the genuine propositions whose truth-values are fixed.

To see how SSSC works, consider this example: **Example 3**. Let *F* be the tensed proposition that sparrows are flying away from Washington, DC today, let *R* be that animals are running away from Washington, DC today, and let *Q* be that there is an earthquake in Washington, DC tomorrow. On Sunday, Jane's credence in [*Q*'s truth on Tuesday] given [*F*'s truth on Monday]&[*R*'s truth on Tuesday] is 0.3. On Tuesday, she remembers that she observed sparrows flying away from Washington, DC and that it was Monday, and she is sure that she is observing animals running away from Washington, DC and it is Tuesday. Jane did not have any other relevant evidence from Monday to Tuesday. In this case, to what degree should Jane believe in *Q* on Tuesday?

Intuitively, her credence on Tuesday in *Q* must be 0.3. For she believed that [*Q* would be true on Tuesday] to the degree of 0.3 conditional on the assumption that [*F* would be true on Monday] and [*R* would be true on Tuesday], and her observations on Monday and Tuesday exactly confirm this assumption. SSSC captures this intuition, as it is an instance of SSSC for this example that C_{TUE} (Q)=C_{SUN} (Q in v_T/(F in v_M)&(R in v_T))=0.3, where v_M is Monday and v_T is Tuesday and C_{SUN} and C_{TUE} are Jane's credence functions on Sunday and Tuesday.

Next, let's think about how to generalize SSSC for the following cases: The agent updates from her old credence function at t_n to a new credence function at t_{n+m}, but she is unsure what observations she has made and/or what times it has been after the *m* epistemic moments earlier time (which we know to be t_n). In this case, SSSC does not apply because its proviso is not satisfied. So, what is the rational way for the agent to update her credence?

Although SSSC does not apply to such a case, it provides an important clue for the answer: Let $\{\mathcal{E}_o\&\mathcal{V}_o\}_{o\in O}$ be a partition such that $\mathcal{E}_o$=($E^1{}_o$ at prev$_{m-1}$)&...& ($E^m{}_o$ at prev$_0$) and $\mathcal{V}_o$=($V^1{}_o$ at prev$_{m-1}$)&...&($V^m{}_o$ at prev$_0$) for each $o\in O$. I consider each $\mathcal{E}_o\&\mathcal{V}_o$ to represent a possible scenario of what observations an agent B has made at what moments (hereafter: a possible observational scenario). For simplicity, let O be {1, 2, ...p}. By SSSC:

$$C_{n+m}(X) \text{ would be } \begin{cases} C_n(X \text{ in } v^m{}_1/\mathcal{D}_1) \text{ if } B \text{ were sure at } t_{n+m} \text{ that } \mathcal{E}_1\&\mathcal{V}_1 \text{ is true,} \\ C_n(X \text{ in } v^m{}_2/\mathcal{D}_2) \text{ if } B \text{ were sure at } t_{n+m} \text{ that } \mathcal{E}_2\&\mathcal{V}_2 \text{ is true,} \\ \quad\ldots \\ C_n(X \text{ in } v^m{}_p/\mathcal{D}_p) \text{ if } B \text{ were sure at } t_{n+m} \text{ that } \mathcal{E}_p\&\mathcal{V}_p \text{ is true,} \end{cases}$$

where $\mathcal{D}_o$=($E^1{}_o$ in $v^1{}_o$)&...&($E^1{}_o$ in $v^1{}_o$) for each $o\in O$ i.e., each $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$. Given these facts, it is natural that B's credence at $t_{\text{n+m}}$ in X is the weighted average of values on the right-hand sides of the above equations with the weights coming from B's credences at t_{n+m} in $\mathcal{E}_o\&\mathcal{V}_o$.

To formalize this idea, we need to have some preliminary jobs done. Consider a partition $\{\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o)$ at prev$_{m-k})\}_{o\in O}$ such that (i) $C_{n+m}((E^k{}_o\&V^k{}_o)$ at prev$_{m-k}$)>0 for each $o\in O$ and (ii) $\sum_{o\in O}C_{n+m}((E^k{}_o\&V^k{}_o)$ at prev$_{m-k}$)=1 where C_{n+m} is an agent B's credence function at t_{n+m}. (The intended interpretation of this partition is that each member is a candidate for the observational scenario that B goes through during $(t_{n+m},t_{n+m}]$.) I will call any member of this partition "(B's) sequential time-observation proposition from t_n to

t_{n+m}." If $\{\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o)$ at $\text{prev}_{m-k})\}_{o\in O}$ also satisfies the condition that for each $o\in O$, $C_n(\&_{1\leq k\leq m}(E^k{}_o$ in $v^k{}_o))>0$, then I will call the partition "(B's) sequential time-observation partition from t_n to t_{n+m}."

Now, I am ready to formulate my next updating principle, "Sequential Shifted Jeffrey Conditionalization": Let $\{\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o)$ at $\text{prev}_{m-k})\}_{o\in O}$ be an agent B's sequential time observation partition from t_n to t_{n+m}. Let C_n and C_{n+m} be B's credence functions at t_n and t_{n+m}. Assume that (#) the truth-value of X is invariant within $v^m{}_o$ and that of $E^k{}_o$ is invariant within $v^k{}_o$, for any $k\in\{1,...,m\}$ and $o\in O$. Then,

(SSJC) $C_{n+m}(X)=$

$$\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o))C_{n+m}(\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k})),$$

where C_n and C_{n+m} are B's credence functions at t_n and t_{n+m}. Less formally: For each $o\in O$, $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k{}_o$ at $\text{prev}_{m-k})$ and $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o$ at $\text{prev}_{m-k})$. Clearly, $\{\mathcal{E}_o\&\mathcal{V}_o\}_{o\in O}=\{\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o)$ at $\text{prev}_{m-k})\}_{o\in O}$. Assume that (#) is satisfied. Then,

(SSJC) $C_{n+m}(X)$=the weighted average of C_n(the de-indexicalization of X under $V^m{}_o$/the sequential de-indexicalization $\mathcal{E}_o$ under $\mathcal{V}_o$) with the weights coming from $C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o)$,

where C_n and C_{n+m} are B's credence functions at t_n and t_{n+m}. I believe this is a natural generalization of SSSC for when the agent is not sure of what sequence of observations she has made at what times.

To understand how SSJC works, think about this example: **Example 4**. Let F be the tensed proposition that sparrows are flying away from Washington, DC today, R_{FROM} be that animals are running from Washington, DC today, R_{TO} be that animals are running to Washington, DC today, and Q be that there is an earthquake in Washington, DC tomorrow. On Sunday, Jane's credence in [Q's truth on Tuesday] given [F's truth on Monday] & [R_{FROM}'s truth on Tuesday] was 0.8, and her credence in [Q's truth on Tuesday] given [F's truth on Monday]&[R_{TO}'s truth on Tuesday] was 0.4. On Monday, she observes that sparrows are flying away from Washington, DC and is certain of what she is observing. On Tuesday, she observes that animals are running but is uncertain whether they are running *from* Washington, DC or *to* Washington, DC. She observes nothing else during the two days. On either day, she knows what day it is.

As a result, she is certain on Tuesday that she previously observed F and it was Monday, and she is also certain on Tuesday that it is presently Tuesday. However, she is uncertain about whether she is observing R_{AWAY} or R_{BACK}. Indeed, her credence on Tuesday in [F's previous truth and R_{FROM}'s present truth]=0.5=her credence on Tuesday in [F's previous truth and R_{TO}'s present truth]. I suggest that her credence on Tuesday in Q will be 0.6. Look at Figure 6:

Time (from our point of view)	**Monday**	**Tuesday**	
Time (Jane's point of view on Tuesday)	**prev_1**	**prev_0**	
Jane is hesitant on Tuesday between these two sequential time-evidence propositions.	***F&MON***	***R_{AWAY}&TUE***	**} The first scenario**
	F&MON	***R_{BACK}&TUE***	**} The second scenario**

Figure 6: Evidential Uncertainty in Sequential Updating. The two rows represent two possibilities about what Jane has observed on what days, during the last two days.

In this figure, the two rows dubbed "the first scenario" and "the second scenario" represent the observational scenarios Jane might have gone through from Monday to Tuesday. (i) On Sunday, Jane's credence in *Q*'s truth on Tuesday was 0.8 given that [*F* would be true on Monday and R_{FROM} would be true on Tuesday]. On Tuesday, if *F&MON* was previously true and R_{FROM}&*TUE* is presently true, it will confirm the bracketed condition. Therefore, Jane's rational credence on Tuesday in *Q* is 0.8 conditional on *F&MON*'s previous truth and R_{FROM}'s present truth. (ii) Similarly, Jane's credence on Tuesday in *Q* will be 0.4 conditional on *F&MON*'s previous truth and R_{TO}'s present truth. (iii) Therefore, her rational credence on Tuesday in *Q* is 0.6, the average of 0.8 and 0.4. I find this line of reasoning to be intuitive.

Interestingly, SSJC supports this intuitive claim. For it is an instance of SSJC that $C_{TUE}(Q)=C_{SUN}(Q \text{ in } v_T/(F \text{ in } v_M)\&(R_{FROM} \text{ in } v_T))*C_{TUE}((F\&V_M \text{ at prev}_1)\& (R_{FROM}\&V_T \text{ at pres}))+C_{SUN}(Q \text{ in } v_T/(F \text{ in } v_M)\&(R_{TO} \text{ in } v_T))*C_{TUE}((F\&V_M \text{ at prev}_1)\& (R_{TO}\&V_T \text{ at pres}))=0.6$, where v_M is Monday and v_T is Tuesday and C_{SUN} and C_{TUE} are Jane's credence functions on Sunday and Tuesday.

So far, so good. In the above example, the agent is uncertain of what she has observed. But what if she is uncertain about what times it has been when she made those observations? Consider this case: **Example 5**. Keep the meanings of "*F*," "R_{FROM}," and "*Q*" the same. On Sunday, Jane's credence in [*Q*'s truth on Tuesday] given [*F*'s truth on Monday]&[R_{TO}'s truth on Tuesday] is 0.8, and her credence in [*Q*'s truth on Wednesday] given [*F*'s truth on Tuesday]& [R_{TO}'s truth on Wednesday] is 0.4. On that night, she takes sleeping pills, but she realizes that she might have overdosed. If she did, she will wake up on Tuesday. (Indeed, she didn't overdose and will wake up and observe *F* on Monday and R_{TO} on Tuesday.)

On Monday, she wakes up and observes *F*. Then, she is immediately put to sleep again, without taking any sleeping pill. (So she expects to wake up on the next day; for the same reason, when she wakes up on the next day, she knows that only one day has passed.) Waking up on Tuesday, Jane certainly knows that she previously observed *F* and is presently observing R_{TO}. However, since she is not sure that she didn't overdose, she is not sure that [it was previously Monday and it is presently Tuesday]; as far as she knows, [it might have been Tuesday at the previous moment and it might be Wednesday now]. Consequently, her credence on Tuesday that [it was previously Monday & it's Tuesday now]=0.5=her credence on Tuesday that [it was previously Tuesday & it's Wednesday now].

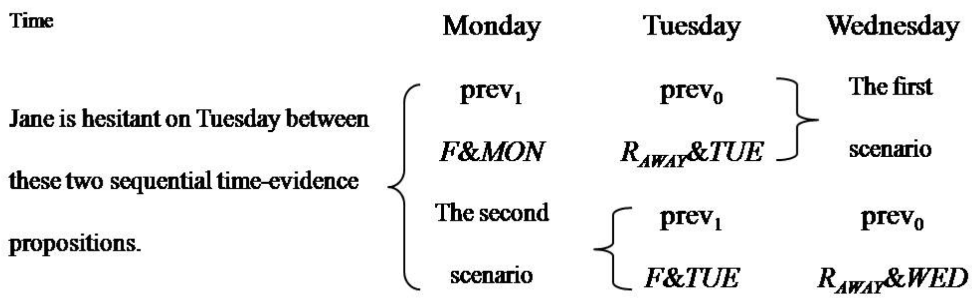

Figure 7: Temporal Uncertainty in Sequential Updating. The two scenarios represent two possibilities about what observations Jane has made on what days, for the last two days.

I claim that Jane's credence on Tuesday in Q is 0.6. Why? Look at Figure 7: In this figure, the two pairs of rows dubbed "the first scenario" and "the second scenario" represent observational scenarios Jane might have gone through. (i) On Sunday, Jane's credence in Q's truth on Tuesday was 0.8 given that [F would be true on Monday and R_{TO} would be true on Tuesday]. On Tuesday, if $F\&MON$ was previously true and $R_{TO}\&TUE$ is presently true, it will confirm the bracketed condition. Intuitively, her rational credence on Tuesday in Q is 0.8 given that $F\&MON$ was previously true and $R_{TO}\&TUE$ is presently true. (ii) Similarly, Jane's rational credence on Tuesday in Q is 0.4 given that $F\&TUE$ was previously true and $R_{TO}\&WED$ is presently true. (iii) Therefore, her credence on Tuesday in Q is 0.6, the average of 0.8 and 0.4. I find this to be intuitive reasoning.

Importantly, SSJC supports this intuitive claim, as it is an instance of SSJC that $C_{TUE}(E)=C_{SUN}(E \text{ in } v_T/(F \text{ in } v_M)\&(R_{TO} \text{ in } v_T))*C_{TUE}((F\&V_M \text{ at prev}_1)\&(R_{TO}\&V_T \text{ at pres}))+C_{SUN}(E \text{ in } v_W/(F \text{ in } v_T)\&(R_{TO} \text{ in } v_W))*C_{TUE}((F\&V_T \text{ at prev}_1)\&(R_{TO}\&V_W \text{ at pres}))=0.6$, where v_M, v_T, and v_W are Monday, Tuesday, and Wednesday, and C_{SUN} and C_{TUE} are Jane's credence functions on Sunday and Tuesday.

Finally, I want to provide more succinct formulations of SSSC and SSJC. For, while the earlier formulations were good for the purpose of explanation, they often will be too bulky for other purposes. So: Let $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k_o \text{ at prev}_{m-k})$ and $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k_o \text{ at prev}_{m-k})$.[37] Given this partition, we can define these notions: (i) $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ is *logically self-optimal* iff for each $o\in O$ and $k\in\{1,...,m\}$, the truth-value of E^k_o is invariant within v^k_o, (ii) $\{\mathcal{V}_o\}_{o\in O}$ is *logically optimal for X* iff for each $o\in O$, the truth-value of X is invariant within v^m_o, and $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ is *logically optimal for X* iff $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ is self-optimal and $\{\mathcal{V}_o\}_{o\in O}$ is optimal for X. Then,

(SSJC) $C_{n+m}(X)=\Sigma_{o\in O}C_n(X \text{ in } v^m_o/\mathcal{D}_o)C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o)$ if $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ is logically optimal for X,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ for each $o\in O$. Next, let $\{\mathcal{E} \& \mathcal{V}\}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E} = \&_{1\leq k\leq m}(E^k \text{ at prev}_{m-k})$ and $\mathcal{V} = \&_{1\leq k\leq m}(V^k \text{ at prev}_{m-k})$. Then,

(SSSC) $C_{n+m}(X)=C_n(X \text{ in } v^m/\mathcal{D})$ if $\{\mathcal{E} \& \mathcal{V}\}$ is logically optimal for X,

[37] Note that $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O} = \{\&_{1\leq k\leq m}(E^k_o \& V^k_o \text{ at prev}_{m-k})\}_{o\in O}$. By the definition of sequential time-observation partition, $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ is such that (i) $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k_o \text{ at prev}_{m-k})$ and $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k_o \text{ at prev}_{m-k})$, (ii) for each $o\in O$, $E^1_o,\ldots, E^m_o$ are the candidates for B's observations at $t_1,...,t_{n+m}$, (iii) for each $o\in O$, $v^1_o,\ldots, v^m_o$ are the candidates for B's temporal locations at $t_1,...,t_{n+m}$, (iv) $C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o)>0$ for each $o\in O$ and $\sum_{o\in O}C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o)=1$, and (v) for each $o\in O$, $C_{n+m}(\mathcal{D}_o)>0$, where $\mathcal{D}_o = \&_{1\leq k\leq m}(E^k_o \text{ in } v^k_o)$.

where $\mathcal{D}$ is the sequential de-indexicalization of $\mathcal{E}$ under $\mathcal{V}$. Clearly, these reformulations are equivalent to the original.

In this section, I discussed two sequential *de nunc* updating principles, SSSC and SSJC. Just as SSC and SJC subsume the probabilistic inferential patterns we find to be intuitive, SSSC and SSJC also subsume intuitive probabilistic reasoning patterns. In the next section, I will defend these new principles by a modified version of Gaifman's Expert Principle.

E. A Defense of SSJC

In this section, I defend an intuitive principle that I call SSR. Since SSR entails SSJC and SSSC, this will show that SSSC and SSJC are true of the cases similar to the example given in this section.

First, consider this principle, which I call "Shifted Sequential Rigidity": Let $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k_o \text{ at prev}_{m-k})$ and $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k_o \text{ at prev}_{m-k})$. Suppose that $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ is logically optimal for X. For each $o\in O$,

(SSR) $$C_{n+m}(X/\mathcal{E}_o \& \mathcal{V}_o) = C_n(X \text{ in } v^m_o/\mathcal{D}_o),$$

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ for each $o\in O$. Here, $\mathcal{E}_o \& \mathcal{V}_o$ codifies what observations B has made in what times, and B knows that $\mathcal{D}_o$ is equivalent to $\mathcal{E}_o$ if $\mathcal{V}_o$ is true. It is not difficult to see that SSR entails SSJC.[38] Hence, it suffices to defend SSR.

But how do we defend it? Think about this principle, which I will call "Sequential Shifted Backward Reflection": Again, let $\{\mathcal{E}_o\&\mathcal{V}_o\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k{}_o$ at $\text{prev}_{m-k})$ and $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o$ at $\text{prev}_{m-k})$. Then,

(SSBR) $\quad C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)=r)=r$ if defined,

where $\mathcal{D}_o$ is the sequential de-indexicalization of X under $V^m{}_o$. Approximately, SSBR is the claim that it is rational for B to set her present credence in X to be the same as her earlier credence in the *de-indexicalization* of X conditional on the *sequential de-indexicalization* of $\mathcal{E}_o$. Remember that it is standard to presuppose that the given agent remembers her past credence functions with perfect confidence and correctness. Under this presupposition, SSBR entails SSR.[39]

In sum, SSBR entails SSR, and SSR entails SSJC. Hence, it suffices to defend SSBR. To do so, I appeal to a more general principle, which I call "the Tensed

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[38] Let $\{\mathcal{E}_o\&\mathcal{V}_o\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k{}_o$ at $\text{prev}_{m-k})$ and $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o$ at $\text{prev}_{m-k})$. Suppose that SSR is true, i.e., $C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o)=C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)$ for each $o\in O$, where $\mathcal{D}_o=\&_{1\leq k\leq m}(E^k{}_o$ at $v^m{}_o)$. Then, $C_{n+m}(X)=\Sigma_{o\in O}C_{n+m}(X\&\mathcal{E}_o\&\mathcal{V}_o)=\Sigma_{o\in O}C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o)$ $C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o)=$ (by supposition) $\Sigma_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o)$. Done.

[39] Suppose that SSBR is true, i.e., $C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)=r)=r$ for any $r\in[0,1]$ and $o\in O$. Let $r=C_n(X \text{ in } v_j/\mathcal{D}_o)$. By presupposition, $C_{n+m}(C_n(X \text{ in } v_j/\mathcal{D}_o)=r)=1$. Thus, $C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o)=C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)=r)=$ (by supposition) $r=C_{n+m}(C_n(X \text{ in } v_j/\mathcal{D}_o)=r)$. Done.

Conditional Expert Principle": For any tensed propositions X and E, any genuine propositions X' and E', and any temporal hypothesis V,

(TCE) $C(X/E\&V\&pr_t(X'/E')=r)=r$ if defined and these conditions are met:

(a) C is an agent B's present credence function and pr_t is an agent Ex's credence function at t on which B depends to decide her present credal opinion,

(b) Ex did not know at t whether E' was true, B can presently access all information that Ex had at t, and B has observed no other information possibly except E.

(c) B presently knows that if V is true, [X is presently true iff X' is true at t] and [E is presently true iff E' is true at t].

Suppose (a)-(c). By (a), it seems rational for B to have her credence in X somehow coordinated with Ex's credence distribution. But what is the rational way to do so?

To answer, think about the facts that follow from the following conditions (under (a)-(c)): (i) E is presently true, (ii) V is presently true, and (iii) Ex's credence at t in X' given E' is r. It follows that E' is true, which Ex did not know at t. Given this informational impoverishment, B cannot rationally depend upon Ex's *unconditional* credences at t to set her present credences, conditional on (i)-(iii). Nevertheless, it is still plausible that it is rational for B to use Ex's credences *conditioned upon* E'. For whether E is true is the only information that B presently has but that Ex might have lacked at t, and E's present truth is equivalent to E''s truth at t. Hence, it will be rational for B to use

(iii) to set her present credence. Given this reasoning, it is rational that *B*'s present credence in *X* is *r* on the condition that (i)-(iii) are true.

I claim that SSBR is a sub-principle of TCE. Consider the following instance of SSBR, which involves *B*'s credal transition from t_n to t_{n+m}: Let $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$, $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$, and $\mathcal{D}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)$. So $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$. Then,

$$\text{(20)} \qquad C_{n+m}(X/\mathcal{E}_o \& \mathcal{V}_o \& C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)=r)=r \text{ if defined.}$$

First, it is inevitable for *B* to set her present credence by depending upon her earlier credence distribution. Second, $\mathcal{E}_o$, if true, represents *B*'s total observation during $(t_n, t_{n+m}]$, which *B* could not access at t_n. Third, if $\mathcal{V}_o$ $(\supset V^m{}_o)$ is true, *X* is true iff [*X* in $v^m{}_o$] is true, and $\mathcal{E}_o$ is true iff $\mathcal{D}_o$ is true. Hence, *B* satisfies the provisos of TCE with respect to her credence distribution at t_{n+m}. Therefore, SSBR follows from TCE as a special case.

Consider this example: **Example 6**. Keep the meanings of "*F*," "R_{FROM}," and "*Q*" the same as in the earlier examples. At 9:00 AM on Tuesday (hereafter: *t*), Jane regards herself at 9:00 PM on Sunday (hereafter: *s*) as an expert about local natural phenomena. Then, what should her credence be at *t* in *Q*, given that (i) ($\mathcal{E}$) *F* was previously true and R_{FROM} is presently true, (ii) ($\mathcal{V}$) it was Monday previously and it is Tuesday now, and (iii) her credence at *s* in *Q*'s truth on Tuesday was 0.7 given that *F* would be true on Monday and R_{FROM} would be true on Tuesday? My answer: it must be 0.7. Formally, $C_t(Q/\mathcal{E}\&\mathcal{V}\&C_s(Q \text{ on Tuesday}/\mathcal{D})=0.7)=0.7$, where $\mathcal{D}$ is the de-indexicalization of $\mathcal{E}$ under $\mathcal{V}$, i.e. the genuine proposition that F is true on Monday and R_{FROM} is true on Tuesday.

Why? Consider Figure 8:

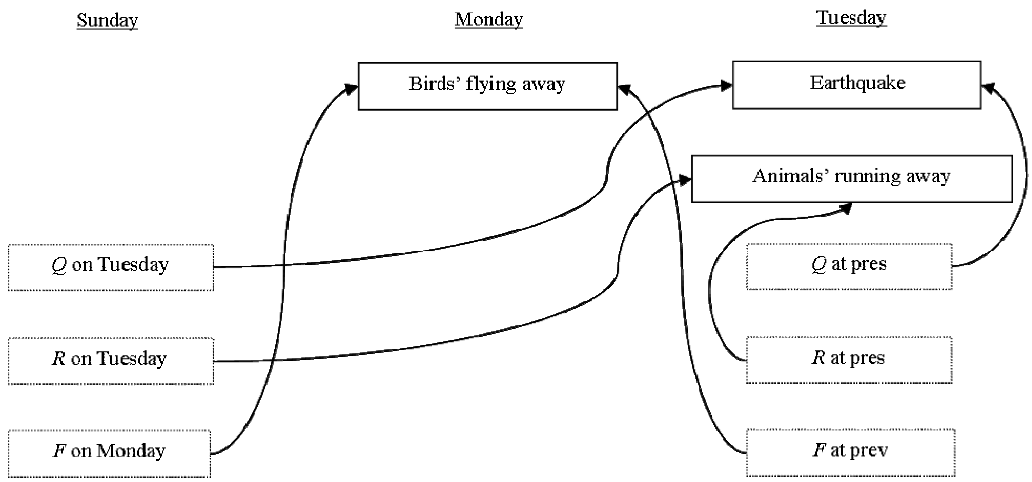

Figure 8: Flying Birds, Running Animals, and Earthquake. The belief at the tail of an arrow is true on the day of the belief's column iff the event at the head of the arrow occurs on the day of the event's column.

First, to set her credence at t in Q, Jane seems to have no other choice but to use her earlier credence distributions, possibly that at s. However, second, she must not simply adopt her unconditional credence at s in Q as her credence at t in Q. This is because Jane has observed some facts relevant to Q, such as birds flying away and animals running away, after s. So what is the rational way for Jane to set her credence at t in Q by utilizing her credence distribution at s? Third, here is a suggestion: Let $\mathcal{E}$ be (F at prev)&(R at pres) and $\mathcal{V}$ be (MON at prev)&(TUE at pres). Then, Jane's credence at t in Q is r on the assumption that (i) $\mathcal{E}$ is presently true, (ii) $\mathcal{V}$ is presently true, and (iii) her credence at s in

[Q on Tuesday] is r given $\mathcal{D}$, where $\mathcal{D}$=(F on Monday)&(R_{FROM} on Tuesday). For although Jane cannot trust her credal judgments on Sunday for the reason mentioned, she can still trust those conditioned upon $\mathcal{D}$, which codifies her observations after Sunday. Plus, Jane knows on Tuesday that if $\mathcal{V}$ is true, then Q is equivalent to [Q on Tuesday], and so she can rationally set her credence in Q by checking her credence on Sunday in [Q on Tuesday] conditioned upon $\mathcal{D}$. (We need to substitute "must" for "can" in the last sentence in light of the outdated conditional credence problem, which we discussed in Section B.)

We can generalize this reasoning: $C_{n+m}(X/\mathcal{E}_{\text{o}}\&\mathcal{V}_{\text{o}}\&C_n(X \text{ in } v^m{}_o/\mathcal{D}_{\text{o}})=r)=r$ where $\mathcal{E}_{\text{o}}=\&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$ is possibly the agent B's sequential total observation during $(t_n,t_{n+m}]$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$ is a temporal hypothesis about when B has made observations during $(t_n,t_{n+m}]$, and $\mathcal{D}_{\text{o}}=\&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)$ is the sequential de-indexicalization of $\mathcal{E}_{\text{o}}$ under $\mathcal{V}_o$. It is easy to see that at t_{n+m}, [X in $v^m{}_o$] and $\mathcal{D}_{\text{o}}$ are genuine propositions equivalent to X and $\mathcal{D}_{\text{o}}$ respectively. Hence, it is intuitively rational at t_{n+m} for B to defer to herself at t_n in the suggested way. It follows that SSBR is true.

So far, I have argued that (i) since an agent at t_{n+m} will usually regard herself at t_n as an expert only lacking the information of what sequence of observations she would make, SSBR is the correct method of deference to her past self, (ii) under the usual presupposition of perfect memory, SSR follows from SSBR, and (iii) since SSR entails SSJC, SSJC and SSSC are the correct rules for *de nunc* updating.

However, I suspect that the reasoning provided in this section doesn't justify all instances of SSSC or SSJC. In the next section, I will apply SSSC to the SB problem and show that it yields an inconsistent result. In Section G, I will try to identify the source of the problem and how to fix SSSC and SSJC.

F. The SB Problem and the Inconsistency of SSJC

In this section, we apply SSSC to the SB problem. Interestingly, it will be shown that if SB updates her credence in *H* in accordance with SSSC, it leads to an inconsistent result.

Remember the temporal structure of the SB problem:

(*s*) SB is put to sleep on Sunday.

(*m*) SB wakes up on Monday.

(*m*+) SB is told that it is Monday.

We can think about three credal transitions: (i) the transition from *s* to *m*, (ii) that from *m* to *m*+, and (iii) that from *s* to *m*+. There are two possible strategies for SB to update from *s* to *m*+: (a) updating from *s* to *m*+ all at once or (b) updating from *s* to *m* and *m* to *m*+ step by step. This raises a concern: What if the results of (a) and (b) do not match?

Unfortunately, this concern is legitimate. First, consider this instance of SSSC, for the updating from *m* to *m*+:

(21) $C_{m+}(H)=C_m(H \text{ on Monday}/MON \text{ on Monday})=C_m(H)<1/2.$

This is a correct instance of SSSC because SB is sure at *m*+ of *MON*&*MON* (the conjunction of her observation proposition and the interval information, which happen to both be *MON* in this case). First, [*MON* on Monday] is redundant because “it’s Monday” is trivially true on Monday. Second, [*H* on Monday] is equivalent to *H* because *H* is a genuine proposition whose truth-value is insensitive to time. Thus, $C_{m+}(H)=C_m(H)$. However, in the previous chapter, we already determined that $C_m(H)<1/2$.

Next, consider this instance of SSSC for the updating from *s* to *m*+:

(22) $C_{m+}(H)=C_s(H \text{ on Monday}/W \text{ on Monday } \& \ MON \text{ on Monday})=1/2.$

This is a legitimate instance of SSSC because SB is sure at *m*+ that *W*&*MON* was previously true and *MON*&*MON* is presently true. In (22), [*W* on Monday & *MON* on Monday] is redundant because SB knew that she would wake up on Monday and "it's Monday" would be true on Monday. Also, [*H* on Monday] is just the same as *H*. Hence, $C_{m+}(H)=C_s(H)=1/2$.

Obviously, (21) and (22) are mutually inconsistent. Since both are instances of SSSC, and so of SSJC, this means that they are inconsistent updating rules.

G. A Diagnosis and a Potential Solution

So SSSC (which is a special case of SSJC) is false because it leads to mutually inconsistent results. Does this mean that we should totally abandon SSSC and SSJC? No. In this section, I discuss Elga's view that temporal knowledge is an essential element of expertise, and I argue that we have to modify SSSC and SSJC slightly so that the modified rules do not apply to the cases where the agent suffers from temporal ignorance. Fortunately, this modification enables us to avoid the aforementioned inconsistency.

Elga (2007) argues that a rational agent doesn't have to obey Reflection when she expects herself to suffer from temporal ignorance:[40]

There is another sort of information loss, a sort associated with losing track of … what time it is. Information loss of that sort can also lead to violations of Reflection. For example, suppose that you are waiting for a train. You are only 50% confident that the train will ever arrive, but you are sure that if it does arrive, it will arrive in exactly one hour. Since you have no watch, when fifty-five minutes have in fact elapsed you will be unsure whether an hour has elapsed. As a result, at that time you will have reduced confidence—say, only 40% confidence—that the train will arrive. So at the start, you can be sure that when fifty-five minutes have elapsed, your probability that the train will ever arrive will have gone down to 40%. So your anticipated imperfect ability to keep track of time creates a violation of Reflection. (Elga 2007, 482)

Let A be the proposition that the train arrives at some time, let C_{INIT} be the agent's credence function at the initial moment, and let $C_{55\ MIN+}$ be her credence function in fifty-five minutes. In the above example, it is an instance of Reflection that $C_{INIT}(A/C_{55\ MIN+}(A)=0.4)=0.4$, but $C_{INIT}(A)=C_{INIT}(A/C_{55\ MIN+}(A)=0.4)=0.5$. Elga claims that this violation is understandable because Reflection is a special case of Gaifman's Expert Principle and the agent, at the initial time, must not regard herself in fifty-five minutes as an expert given the expected loss of the track of what time it is.

I find Elga's claim to be plausible. After all, the agent initially knows that in fifty-five minutes, only fifty-five minutes will have passed since the initial moment, but,

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[40] Reflection is the principle that for any proposition X and any real number r, $C_n(X/C_{n+m}(X)=r)=r$ if defined, where C_n and C_{n+m} are agent B's credence functions at t_n and t_{n+m}. Also see the previous chapter for its relation to Gaifman's Expert.

when fifty-five minutes have actually passed, the agent cannot know it because of her temporal ignorance. Thus, there seems to be some information that she can access initially but that she cannot access in fifty-five minutes. This relative ignorance makes it irrational for the agent's initial self to defer to her future self in fifty-five minutes as recommended by Reflection.

If temporal knowledge is an essential element of expertise required by Reflection, it is plausible that temporal knowledge is also a necessary condition of expertise required by Shifted Backward Reflection. To see this point more clearly, think about the following variant of the SB problem: **SB problem 2**. On Sunday, SB knows that she will go through the following experiment: On that night, evil experimenters will put her to sleep and toss a fair coin. Case 1: (*H*) The coin lands heads. Then, she is awakened once in a room with a big electronic calendar. The calendar is slightly faulty because it has a 0.2 chance of showing the day of tomorrow. Case 2: (*T*) The coin lands tails. In this case, SB is awakened in the same room twice, the first time on Monday and the second time on Tuesday. Plus, the experimenters inject her with a drug with the effect of erasing her memory of Monday at some time between her two awakenings. In either case, one minute after she wakes up on Monday or Tuesday, a completely reliable person tells her what day it is.

As before, let *s* be the last conscious moment on Sunday, let *m* be the moment of wakeup on Monday, and let *m*+ be the moment of being told that it's Monday. Also, let W_{TUE} be the tensed proposition expressed by "SB wakes up today watching the calendar displaying 'TODAY IS TUESDAY.'" As a matter of fact, when she wakes up on Monday, SB receives W_{TUE} as evidence.

Let's think about SB's (i) credal transition from *s* to *m*, (ii) that from *m* to *m*+, and (iii) that from *s* to *m*+. If we apply SSJC to (i)-(iii), what will it say about her credences at *m* and *m*+ in *H*?

First, we get the following result by applying SJC (which is a special case of SSJC) to SB's credal transition from *s* to *m*:

(23) $C_m(H)=C_s(H \text{ on Monday}/W_{TUE} \text{ on Monday})C_m(W_{TUE}\&MON)+$
$C_s(H \text{ on Tuesday}/W_{TUE} \text{ on Tuesday})C_m(W_{TUE}\&TUE)$.

Which message the calendar shows on Monday is clearly irrelevant to the result of the coin toss, and so the first conditional credence is 1/2. Whatever the calendar displays on Tuesday, waking up on Tuesday entails the coin's landing tails, and so the second conditional credence is 0. Hence,

(24) $C_m(H)=1/2C_m(W_{TUE}\&MON)$.

Since SB is sure at *m* that she is waking up reading "TODAY IS TUESDAY" on the calendar,

(25) $C_m(H)=1/2C_m(MON/W_{TUE})$.

However, the calendar has a 0.2 chance of displaying the day of tomorrow. Thus, if it is displaying "TODAY IS TUESDAY," it has a 0.8 chance of displaying today's date, in which case today is of course Tuesday, but it also has a 0.2 chance of displaying tomorrow's date, in which case today is Monday. Intuitively, her credence at *m* in *MON* is 0.2 given W_{TUE} as evidence. Therefore,

(26) $C_m(H)=0.1.$

Second, we get the following result by applying SSC (which is a special case of SSSC and therefore of SSJC where the length of the sequence of evidence is 1) to SB's credal transition from *m* to *m*+:

(27) $C_{m+}(H)=C_m(H \text{ on Monday}/MON \text{ on Monday})=C_m(H).$

For SB is told at *m*+ that it is Monday, but it is no secret that *MON* is certainly true on Monday.

Third, we get this result by applying SSSC (which is a special case of SSJC) to SB's credal transition from *s* to *m*+:

(28) $C_{m+}(H)=C_s(H \text{ on Monday}/MON \text{ on Monday } \& W_{TUE} \text{ on Monday})$
$=C_s(H)=1/2.$

For she knew at *s* that *MON* would be true on Monday and, whether the electronic calendar works correctly or not, it is irrelevant to whether the coin lands on heads.

Of course, (26)-(28) are jointly inconsistent; if they are all true, SB's credence in *H* is both 0.1 and 1/2 at the same time. Thus, just as in **SB problem 1**, SSJC leads to inconsistency in **SB problem 2**. The difference is that in the latter case, it is easier to see *which result* of its application is wrong *for what reason*: (27) is false because SB doesn't know at *m* what time it is and SSJC doesn't work correctly when it is used for the credal transition from a moment of temporal ignorance.

To understand why, remember that I argued for SSJC on the basis of SSBR. For my present purpose, it is easier to talk directly in terms of SSBR. Hence, see these instances of SSBR for **SB problem 2**:

(29) $C_m(H/W_{TUE}\&MON\&C_s(H \text{ on Monday}/W_{TUE} \text{ on Monday})=1/2)=1/2$.

(30) $C_m(H/W_{TUE}\&TUE\&C_s(H \text{ on Tuesday}/W_{TUE} \text{ on Tuesday})=0)=0$.

(31) $C_{m+}(H/MON\&MON\&C_m(H \text{ on Monday}/MON \text{ on Monday})=r)=r$, where $r=C_m(H)$.

(32) $C_{m+}(H/[W_{TUE} \text{ at prev } \& \ MON \text{ at pres}]\&[MON \text{ at prev } \& \ MON \text{ at pres}] \ \& \ C_s(H \text{ on Monday}/MON \text{ on Monday } \& \ W_{TUE} \text{ on Monday})=1/2)=1/2$.

First, (29) and (30) are the instances of SBR (and therefore of SSBR) for SB's credal transition from *s* to *m*. Under the assumption that she always remembers her past credence functions with perfect correctness and confidence, it follows from them that $C_m(H/W_{TUE}\&MON)=1/2$ and $C_m(H/W_{TUE}\&TUE)=0$, which eventually leads to (26). Second, (31) is an instance of SBR for her updating from *m* to *m*+. Under the same assumption, it leads to (27). Third, similarly, (32) leads to (28). Therefore, there is a contradiction.

The common idea behind these instances of SSBR is that SB must consider herself at a past moment as an expert in a limited sense.[41] For while the agent has no choice but to depend on her past self to set her present credences, she is also aware that she has acquired some new information. Thus, I suggested that, roughly, SB must defer not to her *unconditional* past credence distribution but to her past credence distribution *conditioned upon* the observations that she has made after she has the past credence distribution.

However, this idea fails to support (31). For when she is told that it is Monday, SB realizes that it was due to the *faultiness* of the electronic calendar that she was strongly biased to the possibility that it was Tuesday then. Being aware of this fact, she shouldn't trust her own previous credal judgment, which was based upon that faulty temporal information.

In general, just as a rational agent cannot consider her future self to be an expert when she expects to lose track of time in the future, a rational agent cannot regard her past self to be an expert when she remembers that she lost track of time in the past. For this reason, I believe that SSBR is not true of cases in which the agent previously did not know what time it was. Since SSJC gains its plausibility from SSBR, we should apply SSJC only to cases in which the agent knew what time it was.

Given this diagnosis, let's think about how it affects the three credal transitions in the original SB problem: (i) the credal transition from *s* to *m*, (ii) that from *m* to *m*+,

[41] Elga uses "guru" to refer to an agent who is an expert in this limited sense. See Elga (2007).

and (iii) that from *s* to *m*+. If we apply SSJC to all these credal transitions, we are confronted with a contradiction: By applying SJC to (i), I have shown that SB's credence at *m* in *H* is less than 1/2. By applying SSC to (ii), I have proven that her credence at *m*+ in *H* has the same value as her credence at *m* in *H*. However, by applying SSJC to (iii), I have shown that her credence at *m*+ in *H* is 1/2. Hence, $1/2>C_m(H)=C_{m+}(H)=1/2$. A contradiction.

Fortunately, our discussion in this section suggests that it is faulty to apply SSC to (ii). For look at Figure 9: As you see in this figure, SB didn't know at *m* that it was Monday, having lost track of what time it was. Thus, at *m*+, SB cannot regard herself at *m* as an expert. If SSJC and its sub-principles are correct only when the agent knew what time it was at the time from which she is updating, this means that SSC does not apply correctly to SB's credal transition from *m* to *m*+.

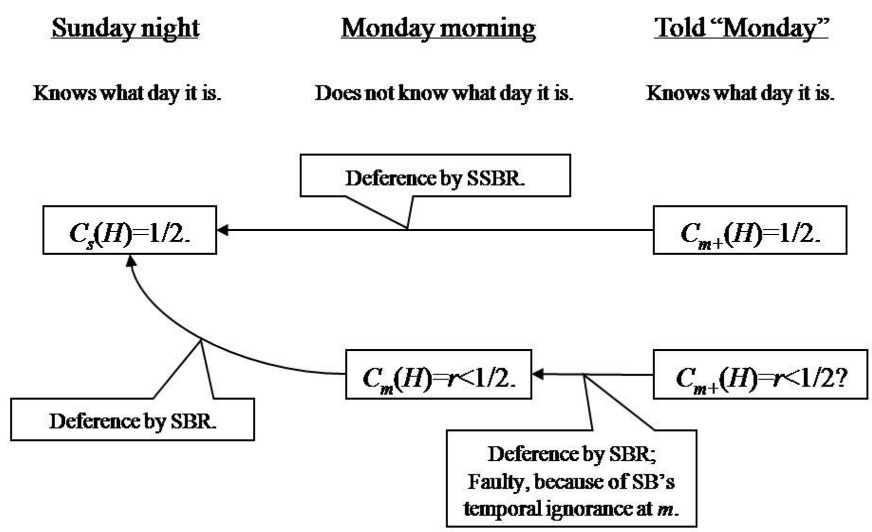

Figure 9: Deference and Temporal Ignorance. It is not necessarily the case that $C_{m+}(H)=C_m(H)=r<1/2$. For it is irrational at m+ *for SB to defer to her previous credal judgment, provided that she suffered from temporal ignorance at* m.

This suggests the following view: When she wakes up on Monday, SB's credence in H is lower than 1/2 by SJC applied to (i), and, when she is told that it is Monday, her credence in H is 1/2 by SSSC applied to (iii). This is exactly the popular thesis of Thirders.

The discussion until now suggests that we have to modify SSSC and SSJC by adding the proviso that to update one's credences from t_n to t_{n+m} using these rules, an agent B must be sure at t_{n+m} that she didn't suffer from temporal ignorance at t_n. Here are the results of this modification, which I call "Restricted Sequential Shifted Jeffrey Conditionalization*" and "Restricted Sequential Shifted Strict Conditionalization*": Let $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k_o \text{ at prev}_{m-k})$ and $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k_o \text{ at prev}_{m-k})$. Then,

(RSSJC*) $C_{n+m}(X)=C_n(X \text{ in } v^m_o/\mathcal{D}_o)C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o)$ if $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ is logically optimal for X and B was free from temporal ignorance at t_n,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ for each $o\in O$. Next, let $\{\mathcal{E} \& \mathcal{V}\}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E} = \&_{1\leq k\leq m}(E^k \text{ at prev}_{m-k})$ and $\mathcal{V} = \&_{1\leq k\leq m}(V^k \text{ at prev}_{m-k})$. Then,

(RSSSC*) $C_{n+m}(X)=C_n(X \text{ in } v^m/\mathcal{D})C_{n+m}(\mathcal{E} \& \mathcal{V})$ if $\{\mathcal{E} \& \mathcal{V}\}$ is logically optimal for X and B was free from temporal ignorance at t_n, where $\mathcal{D}$ is the sequential de-indexicalization of $\mathcal{E}$ under $\mathcal{V}$. Now, (21) and (27), the troublesome instances of SSSC in **SB problems 1&2**, don't follow from RSSSC* because its proviso is violated.

Also note that SSC and SJC are the special cases of SSSC and SSJC, respectively, where the length of sequential timed evidence is 1. Just as we modified the sequential principles into their restricted versions, we can restrict SSC and SJC with the additional proviso that the agent was previously free from temporal ignorance. Let's call the restricted versions of SSC and SJC "Restricted Shifted Strict Conditionalization*" and "Restricted Shifted Jeffrey Conditionalization*". (Since the formal modification is obvious, I do not provide explicit formulation of RSSC* and RSJC* here.)

In this section, I have provided a diagnosis of the problem discussed in Section F and discussed how to modify SSSC, SSJC, SSC, and SJC accordingly. The modification seems successful in removing the aforementioned problem.

H. Conclusion

In this chapter, I have argued for two new principles for sequential updating. I tried to defend those principles, SSSC and SSJC, by a reasoning similar to that which I used for SSC and SJC in Chapter II.

However, SSSC and SSJC turned out to be inconsistent when applied to the three updating paths in the SB problem. With the help of Elga's discussion, I provided a diagnosis for the problem of SSSC and SSJC, demonstrating that they are inconsistent when updating from a credence function when the agent suffers from temporal ignorance. Accordingly, I suggested weaker modifications of these principles, RSSSC* and RSSJC*.

These yet new principles are not obviously inconsistent because they don't apply to cases in which the agent suffers from temporal ignorance.

Seemingly, this leads to a happy ending: We have updating rules that are free from obvious inconsistencies and that are justified by plausible arguments. Nevertheless, I am not perfectly satisfied. On the one hand, the provisos of RSSSC* and RSSJC* are too restrictive. For it is clearly desirable to have updating rules applicable even to a credal transition at the initial time of which the agent *did* suffer from temporal ignorance. On the other hand, those provisos may not be sufficiently restrictive. For although RSSSC* and RSSJC* didn't result in any apparent contradictions in the discussed counterexamples to SSSC and SSJC, there is no guarantee that we will not find other counterexamples in which the restricted versions also lead to contradictions.

In the succeeding chapters, I will pursue the following goals: First, I will formulate and defend general principles for updating *de nunc* credences that are applicable to a credal transition with initial temporal ignorance. Second, I will show that those new principles are free from the problems of SSSC and SSJC that I have discussed in this chapter.

CHAPTER IV

UPDATING WITH *DE PRIORI* INFORMATION

A. Introduction

In the last chapter, I suggested a rule that a rational agent can use to update her *de nunc* credences after making a sequence of observations. Now, we face a new challenge: Suppose that you learn new information about what time it was at an earlier epistemic moment. For example, you wake up without knowing whether today is Monday or Tuesday and, after a while, you newly learn that *it was Monday when you woke up earlier*. In such a case, what will the rational rule be for updating your credence in a tensed proposition?

None of the rules discussed in Chapter III will help you to find the answer: First, SSJC does not apply correctly to any such case. For you can learn *new* information about what time it was at an earlier time *t* only when you did not know "what time it is now" at *t*, and I already discussed the fact that it would be irrational for you to use the rule of SSJC if you are updating from a moment of temporal ignorance. Second, the less general rules discussed in the last section—SSC, SJC, and SSSC—will not apply correctly to such a case because they are the sub-principles of SSJC.[42]

Due to this problem, we need yet another rule for updating. In this chapter, I will suggest that when an agent newly learns information about what time it was at an earlier

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[42] Moreover, RSSJC* and its sub-principles (RSSSC*, RSSC*, and RSJC*) do not apply to such a case because freedom from temporal ignorance is the common proviso of those rules.

epistemic moment (hereafter: *de priori* information), the agent ought to update her *de nunc* credences by following the updating rule that I call "General Shifted Jeffrey Conditionalization" (hereafter: GSJC). In particular, I want to achieve these goals: to defend GSJC and then to illustrate how it works with examples. Finally, I will apply it to SB's credal updating from m to m+, arguing that the resulting credence at m+ in H is 1/2.

B. Strategy

In this chapter, my goal is to find a rule for updating *de nunc* credences. The rule should be applicable to a credal transition from t_n to t_{n+m} even if the agent is ignorant at t_n of what time it is then. To find a clue for such a principle, consider SB's credal transition from m to m+. If SB updates her credences in accordance with SSC,

(1) $C_{m+}(H)=C_m(H/MON \text{ on Monday})=C_m(H)<1/2.$

Here is a rationale for this claim: When told that it is Monday, SB has no choice but to consult her previous credal judgments to set her present credence in H. However, she made those judgments before learning that it is Monday. Hence, SB needs to consult her previous credal judgments conditioned upon *MON* or something equivalent. In a similar case, it is generally better to consult one's previous credal judgments conditioned upon the de-indexicalization of one's present observations than to consult those conditioned upon the present observations themselves.[43] Thus, her credence at m+ in H will be equal to her credence at m in H conditioned upon [*MON* on Monday].

[43] This is because of the so-called outdated conditional credence problem, which I discussed in the last chapter.

In the last chapter, I criticized this rationale: The above rationale assumes that when she is told that it is Monday, SB can regard her previous self as an expert only lacking what she has just learned. However, her previous self cannot be relied upon as an expert even in this limited sense. For she did not previously know what day it was then, and such knowledge is a necessary condition even for the sort of expertise we are talking about.

To find a better way to set her credence at $m+$ in H, we must remember that when SB wakes up on Monday, she is *ignorant* of what time it is then, and when she is told that it is Monday, she comes to *know* something which frees her from her previous ignorance. With this in mind, I pose three questions: First, what are the contents of SB's ignorance and knowledge? Second, what is the logical relation between those contents? Third, given this relation, what is the rational way for SB to update her credence in H from m to $m+$?

Let's focus on the first question. Waking up on Monday, SB is ignorant of the fact that "it is Monday at the present moment," and when she is told that it is Monday, she acquires the knowledge that "it was Monday at the previous moment" by inference. The contents of the ignorance and knowledge are those expressed by the sentences quoted in the last sentence.

This answer is very plausible. For both "the present moment" and "the previous moment" refer to the same moment ($=m$), and so it is easy to see how the later acquirement of knowledge removes the earlier temporal ignorance. Thus we can say: When she is told that it is Monday, SB is aware of this fact:

(2) By being ignorant of *MON*, I previously suffered from temporal ignorance, from which I have been freed by coming to know [*MON* at prev].

In general, if an agent previously suffered from ignorance of *V*, she can be freed from that ignorance by learning [*V* at prev] now.[44]

Now let's focus on the second question. As previously mentioned, when SB wakes up, she is ignorant of the fact that "it is presently Monday," but when told that it is Monday, she comes to know that "it was previously Monday." Between the contents of these ignorance and knowledge, the following logical relation holds: When told that it is Monday, SB knows that

(3) [*MON* at prev] is presently true iff *MON* was previously true.

In general, [*V* at prev] is presently true iff *V* was previously true, where *V* is a tensed proposition specifying what time it is.

Turning to the third question, I claim that the following equation describes a rational way for SB to update her credence in *H* from *m* to *m*+:

(4) $C_{m+}(H)=C_{m+}(H/MON \text{ at prev})=C_m(H/MON)=1/2$.

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[44] It is true at *m*+ that (*) SB is freed from her previous ignorance of *MON* by coming to know *MON*. However, it is not generally true that if an agent previously suffered from the ignorance of *V*, she can be freed from that temporal ignorance by learning *V*. Consider this case: Jake goes to bed not knowing *THU*, but he is freed from that ignorance by coming to know [*THU* at prev] the next morning. In the last sentence, you cannot replace [*THU* at prev] with *THU salva veritate*.

Here is the rationale for this claim: When she is told that it is Monday, SB realizes that it was previously Monday but remembers that she did not know what day it was then. Hence, she will regard her previous self as ignorant of what day it was then. Still, she has no choice but to depend upon her previous credal judgments to set her present credence in *H*. In this situation, she will be better off consulting her previous credal judgments conditioned upon [*MON* at prev] or something equivalent. According to (3), [*MON* at prev] is true iff *MON* was true at the previous moment. So it seems rational for her to set her present credence in *H* by consulting her previous credence function conditioned upon *MON*.

Is this rationale for (4) vulnerable to the same criticism I raised against the rationale for (1)? No. As (2) says, *MON* is the content of SB's temporal ignorance at the moment of her waking up. Since the consulted conditional credence function is conditioned upon *MON*, it is a credal judgment based upon the temporal information that was correct at the mentioned moment.

Hence, I believe that (4) captures the correct updating pattern for SB's credal transition from *m* to *m*+. The remaining job is to incorporate this pattern into a new principle for *de nunc* updating. For this project, I will proceed in the following order: In Section C, I will present a general rule for updating *de nunc* credences. In Sections D and E, I will defend that rule using a yet new variant of the Conditional Expert Principle. In Section F, I shall reformulate the thus defended rule into a more readily usable form. In Section G, I will apply it to SB's credal transitions from *s* to *m*, from *m* to *m*+, and from *s* to *m*+. The results will turn out to be mutually consistent. In Section H, I will discuss the relation between the rules presented in this chapter and those presented in the earlier chapters, such as SSC, SJC, SSSC, and SSJC.

C. Updating with *De Priori* Information

I will begin this section by defining a few important notions related to *information about what time it was*. Next, I will present a new principle for *de nunc* updating, applicable to a credal transition from a moment of temporal ignorance, and illustrate how it works with an example. Finally, I will provide a shorter formulation of that principle, which I will use in later discussions.

To begin, I ask the following questions: Consider an agent *B*'s credal transition from t_n to t_{n+m}. First, at t_{n+m}, how can *B* specify what times it had been until an earlier epistemic moment? Second, how can we (sort of) translate such information from the context of t_{n+m} to the context of t_n? Third, what will the logical relation be between the original and translated pieces of information?

To answer the first question, I introduce the following definition:

(5) Tensed proposition *F* is *de priori* information iff *F* is [*W* at prev_k] for some special tensed proposition *W* and some number $k \geq 1$.

For example, consider this case: **Example 1**. On Sunday night, Jane knows that it is Saturday or Sunday but does not know which. The next morning, her sincere friend Jeff tells her, "The last time you were awake it was Sunday." From this testimony, Jane learns [*SUN* at $\text{prev}_{(1)}$]. By definition, it is *de priori* information.

This notion of *de priori* information can be further generalized:

(6) Tensed proposition $\boldsymbol{\mathcal{W}}$ is sequential *de priori* information iff $\boldsymbol{\mathcal{W}}$=(W^1 at prev_k)&(W^2 at prev_{k+1})&... &(W^n at $\text{prev}_{k+n\text{-}1}$).

For instance, consider this case: **Example 2**. Briefly waking up on Sunday night, Jane knows that either [it is Sunday now and it was Saturday at the previous moment] or [it is Saturday now and it was Friday at the previous moment], but she does not know which. The next morning, Jeff tells her, "The last time you were awake it was Sunday, and when you were awake before that moment it was Saturday." From this testimony, she learns (*SUN* at prev_1)&(*SAT* at prev_2). By definition, it is sequential *de priori* information. As you can easily see, such information is about what time*s* it had been until an earlier epistemic moment.

Next, I introduce a definition for a special case of sequential *de priori* information, namely, information about what times it had *ever* been until an earlier epistemic moment: For simplicity, I suppose that *B* has the first epistemic moment t_0, i.e., *B* received her first evidence at t_0. (It is possible but less elegant to discuss my view without this supposition.) Then,

(7) $\boldsymbol{\mathcal{W}}$ is a temporal description at t_{n+m} of the epistemic moments until t_n iff $\boldsymbol{\mathcal{W}}$=(W^1 at prev_m)&(W^2 at prev_{m+1})&…&(W^{n+1} at prev_{m+n}).

At t_{n+m}, "prev$_{m+n}$" refers to t_0. Hence, the temporal description at t_{n+m} of the epistemic moments until t_n exhaustively specifies what times it had been at the epistemic moments between t_0 and t_n. This answers the first question.

To answer the second question, I introduce this notion:

(8) [W at prev$_{k-m}$] is the re-indexicalization of [W at prev$_k$] for the m epistemic moments earlier time.

Consider **Example 1** again. When Jane learns [*SUN* at prev$_1$] from Jeff, *SUN* (or [*SUN* at prev$_0$]) is its re-indexicalization for the previous epistemic moment. We can generalize this notion for sequential *de priori* information:

(9) $\boldsymbol{\mathcal{R}}$ is the sequential re-indexicalization of $\boldsymbol{\mathcal{W}}$ for the m epistemic moments earlier time iff

(i) $\boldsymbol{\mathcal{W}}$=(W^1 at prev$_k$)&(W^2 at prev$_{k+1}$)&…&(W^n at prev$_{k+n}$) and

(ii) $\boldsymbol{\mathcal{R}}$=(W^1 at prev$_{k-m}$)&(W^2 at prev$_{k+1-m}$)&…&(W^n at prev$_{k+n-m}$).

To understand this definition, consider **Example 2** again. When Jane learns (*SUN* at prev$_1$)&(*SAT* at prev$_2$) from Jeff, (*SUN* at prev$_0$)&(*SAT* at prev$_1$) is its sequential re-indexicalization for the previous moment. In a good sense, (*SUN* at prev$_0$)& (*SAT* at prev$_1$) is the "translation" of (*SUN* at prev$_1$)&(*SAT* at prev$_2$) from Monday morning to Sunday night.

Now, remember that the temporal description at t_{n+m} of the epistemic moments until t_n *exhaustively* specifies what times it had been until t_n. By definition, such a temporal description is sequential *de priori* information, and so it will have a suitable sequential re-indexicalization. Thus, I make this claim: Consider the temporal description $\boldsymbol{\mathcal{W}}$ at t_{n+m} of the epistemic moments until t_n. Then, $\boldsymbol{\mathcal{W}}$'s sequential re-indexicalization $\boldsymbol{\mathcal{R}}$ for the *m* epistemic moments earlier time will be, in a good sense, the translation of $\boldsymbol{\mathcal{W}}$ from t_{n+m} to t_n.

To explain why, we need to answer the third question: Let $\boldsymbol{\mathcal{W}}$ be a temporal description at t_{n+m} of the epistemic moments until t_n, and let $\boldsymbol{\mathcal{R}}$ be its re-indexicalization for the *m* epistemic moments earlier time. Then,

(10) $\boldsymbol{\mathcal{W}}$ is true at t_{n+m} iff $\boldsymbol{\mathcal{R}}$ was true at t_n.

Hence, both $\boldsymbol{\mathcal{W}}$ and $\boldsymbol{\mathcal{R}}$ describe what times it had been until t_n, the former from the point of view at t_{n+m} and the latter from the point of view at t_n.

I have answered all three questions, but I still need one more definition to precisely formulate the rules I am working toward: Let $\boldsymbol{\mathcal{R}}$ and $\boldsymbol{\mathcal{R}}$* be sequential *de priori* information such that $\boldsymbol{\mathcal{R}}=(W^1 \text{ at prev}_k)\&(W^2 \text{ at prev}_{k+1})\&\ldots\ \&(W^n \text{ at prev}_{k+n-1})$ and $\boldsymbol{\mathcal{R}}^*=(W^{*1} \text{ at prev}_k)\&(W^{*2} \text{ at prev}_{k+1})\&\ldots\ \&(W^{*n} \text{ at prev}_{k+n-1})$. In a good sense, $\boldsymbol{\mathcal{R}}$ ascribes w^1 to the *k* epistemic moments earlier time, w^2 to the $k+1$ epistemic moments earlier time, …, w^{n+1} to the $k+n$-1 epistemic moments earlier time; similarly for $\boldsymbol{\mathcal{R}}$*. Then,

(11) $\mathcal{R}^*$ is better *de priori* information than $\mathcal{R}$ iff for any $k \in \{1,...,m\}$, $w^{*k} \subseteq w^k$, and for some $k \in \{1,...,m\}$, $w^{*k} \subset w^k$.

In other words, $\mathcal{R}^*$ is better than $\mathcal{R}$ exactly when $\mathcal{R}^*$ attributes at least equally narrow intervals to the mentioned epistemic moments and $\mathcal{R}^*$ attributes a strictly narrower interval to one of the epistemic moments than $\mathcal{R}$. Let $\mathcal{R}$ be any sequential *de priori* information. Then, for any probability function C and tensed propositions X and Y,

(12) $\mathcal{R}$ is *well-specified de priori information with respect to* $<C, X, Y>$ iff $C(X/Y\&\mathcal{R})=C(X/Y\&\mathcal{R}^*)$ for any sequential *de priori* information $\mathcal{R}^*$ better than $\mathcal{R}$.

When this condition is satisfied, I will often say informally that $C(X/Y\&\mathcal{R})$ is conditioned upon well-specified *de priori* information. This definition allows us to formulate the desired rules without precisely specifying what times it had been until the moment from which updating occurs.

Now, I am ready to formulate my first updating rule in this chapter, "General Shifted Strict Conditionalization": Consider a sequence of observations E^1, E^2, ...E^{m}, a sequence of intervals v^1, v^2, ... v^m, and another sequence of intervals w^1, w^2, ... w^{n+1}. Assume that (a) the truth-value of X is invariant within v^{m} and that of E^k is invariant within v^k for each $k \in \{1,...,m\}$ and (b) $C_n(X \text{ in } v^m/(E^1 \text{ in } v^1)\&... \&(E^m \text{ in } v^m)\&(W^1 \text{ at } \mathrm{prev}_m)\&...\& (W^{n+1} \text{ at } \mathrm{prev}_{m+n}))$ is conditioned upon a well-specified temporal description. Then, for any tensed proposition X, (GSSC) $C_{n+m}(X)=C_n(X \text{ in } v^{\mathrm{m}}/(E^1 \text{ in } v^1)\&...\&(E^{\mathrm{m}} \text{ in } v^{\mathrm{m}})\&(W^1 \text{ at prev}_m)\&...\&\ (W^{n+1}$ at $\text{prev}_{m+n}))$ if

(i) B is sure at t_{n+m} that for each $k\in\{1,...,m\}$, [E^k was/is true and it was/is v^k] m-k epistemic moments ago, and

(ii) B is sure at t_{n+m} that for any $k\in\{1,...,n+1\}$, [it was w^k] m+k-1 epistemic moments ago,

where C_n and C_{n+m} are B's credence functions at t_n and t_{n+m}. Less formally, we can rewrite SSSC in this way: Let $\mathcal{E}$ be (E^1 at $\text{prev}_{\text{m-1}}$)&...&(E^{m} at prev_0), $\mathcal{V}$ be (V^1 at $\text{prev}_{\text{m-1}}$) &...&(V^{m} at prev_0), and $\mathcal{W}$ be (W^1 at prev_m)&(W^2 at prev_{m+1}) &…&(W^{n+1} at prev_{m+n}). Assume that (a) and (b) are true. Then,

(GSSC) $C_{n+m}(X)=C_n$(the de-indexicalization of X under V^{m}/the sequential de-indexicalization of $\mathcal{E}$ under $\mathcal{V}$ & the sequential re-indexicalization of $\mathcal{W}$ for the m epistemic moments earlier time) if B has certainly learned until t_{n+m} that $\mathcal{E}\&\mathcal{V}\&\mathcal{W}$ is true.

Unfortunately, GSSC is a principle with an extremely narrow range of application. In order to use it, we need to know what time it has been since our very first observation. Hence, we are forced to move to our next, more general updating principle. First, let me outline the core idea: Let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be a partition such that $\mathcal{E}_o$=($E^1{}_{\text{o}}$ at $\text{prev}_{\text{m-1}}$)&...& ($E^{\text{m}}{}_{\text{o}}$ at prev_0), $\mathcal{V}_o$=($V^1{}_{\text{o}}$ at $\text{prev}_{\text{m-1}}$)&...&($V^{\text{m}}{}_{\text{o}}$ at prev_0), and $\mathcal{W}_o$=($W^1{}_{\text{o}}$ at

prev_{m})&...& ($W^{\text{n+1}}{}_{\text{o}}$ at prev_0), for each $o\in O$. I consider each $\mathcal{E}_o$&$\mathcal{V}_o$ to represent a possible evidential scenario and each $\mathcal{W}_o$ to represent what times it had been until t_n. For simplicity, let O be {1, 2, ...p}. By GSSC:

$$C_{n+m}(X) \text{ would be } \begin{cases} C_n(X \text{ in } v^m{}_1/\mathcal{D}_1\&\mathcal{R}_1) \text{ if } B \text{ were sure at } t_{n+m} \text{ of } \mathcal{E}_1\&\mathcal{V}_1\&\mathcal{W}_1, \\ C_n(X \text{ in } v^m{}_2/\mathcal{D}_2\&\mathcal{R}_2) \text{ if } B \text{ were sure at } t_{n+m} \text{ of } \mathcal{E}_2\&\mathcal{V}_2\&\mathcal{W}_2, \\ \quad\quad ... \\ C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p) \text{ if } B \text{ were sure at } t_{n+m} \text{ of } \mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p, \end{cases}$$

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the m epistemic moments earlier time for each $o\in O$. If we accept GSSC (at least for its narrow range of application), it is natural that B's credence at $t_{\text{n+m}}$ in X is the weighted average of values on the right-hand sides of the above equations with the weights coming from B's credences at $t_{\text{n+m}}$ in $\mathcal{E}_o$&$\mathcal{V}_o$&$\mathcal{W}_o$.

To implement this idea, we need to finish some formal homework first: Consider a partition $\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})\}_{o\in O}$ such that (i) $C_{n+m}(\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1}))>0$ for each $o\in O$ and (ii) $\sum_{o\in O}C_{n+m}(\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1}))=1$ where C_{n+m} is an agent B's credence function at t_{n+m}. (The intended interpretation of this partition is that each member is a hypothesis about (i) what observations have been made at what times after t_n and (ii) what times it had been until t_n.) I will call any member of this partition

"(B's) general time-observation proposition from t_n to t_{n+m}." If that partition also satisfies the condition that for each $o \in O$, $C_n(\&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1}))>0$, then I will call the partition "(B's) general time-observation partition from t_n to t_{n+m}."

Now, I am ready to present my next updating rule, called "General Shifted Jeffrey Conditionalization": Let $\{\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k}) \;\&\; \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m} over $[t_0,t_{n+m}]$. Assume that for any $o\in O$ (a) the truth-value of X is invariant within $v^m{}_o$ and that of $E^k{}_o$ is invariant within each $v^k{}_o$ for any $k\in\{1,...,m\}$ and (b) $C_{n+m}(\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1}))$ is conditioned upon a well-specified temporal description. Then,

(GSJC) $$C_{n+m}(X)=\Sigma_{o\in O}[C_n(X \text{ in } v^m{}_o/\&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1}))$$

$$C_{n+m}(\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1}))],$$

where C_n and C_{n+m} are B's credence functions at t_n and t_{n+m}. Less formally: Let $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})$ for each $o\in O$, so that $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is the same as $\{\&_{1\leq k\leq m}((E^k{}_o\&V^k{}_o) \text{ at prev}_{m-k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})\}_{o\in O}$, B's general time-observation partition from t_n to t_{n+m}. Assume that for each $o\in O$ (a) and (b) are true. Then, (GSJC) $C_{n+m}(X)$ = the weighted average of C_n(the de-indexicalization of X under $V^m{}_o$/the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ & the sequential re-indexicalization of $\mathcal{W}_o$) with the weights coming from $C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$

where C_n and C_{n+m} are B's credence functions at t_n and t_{n+m}. I think this is a natural generalization of GSSC for when the agent is not sure of what sequence of evidence she has received at what times and what times it had been before receiving any sequence of evidence in consideration.

To see how GSSC and GSJC work, consider the following example: **Example 3**. Let R be the tensed proposition expressed by "it rains today in Boston," and let P be that expressed by "there is a form of precipitation today in Boston." Jane is born on Saturday, knowing that (*SAT*) it is Saturday.

Just after her birth, she falls asleep and then wakes up on Sunday. She learns that (*SUN*) it is Sunday. At this time, Jane assigns the credence of 0.8 to [R on Monday] given [P on Monday]. Immediately after waking up, she takes a sleeping pill that will make her wake up either on Monday or Tuesday, but she will not know which day it is when she wakes up.

In fact, Jane wakes up on Monday. On being awakened, she is told that there is a form of precipitation today. For the mentioned reason, she does not know whether (*MON*) it is Monday or (*TUE*) it is Tuesday; indeed, she assigns the credence of 0.5 to each of *MON* and *TUE*. (Hence, 0.5 is her credence at this moment in *P*&*MON*.) Later on that day, she is told that it is Monday.

For brevity, let *b* be the moment of Jane's birth on Saturday, let *s* be the moment of waking up on Sunday, let *m* be the moment of waking up on Monday, and let *m*+ be the moment of being told that it is Monday. We assume that she does not observe anything between these moments. What, then, is her rational credence at *m*+ in *R*?

First, let's focus on her credal transition from *m* to *m*+. By GSSC,

(13) $C_{m+}(R) = C_m(R$ on Monday/(MON on Monday)&(MON at prev$_0$)&(SUN at prev$_1$)&(SAT at prev$_2$)).

The conditional part of the right-hand side was acquired as in Figure 10:

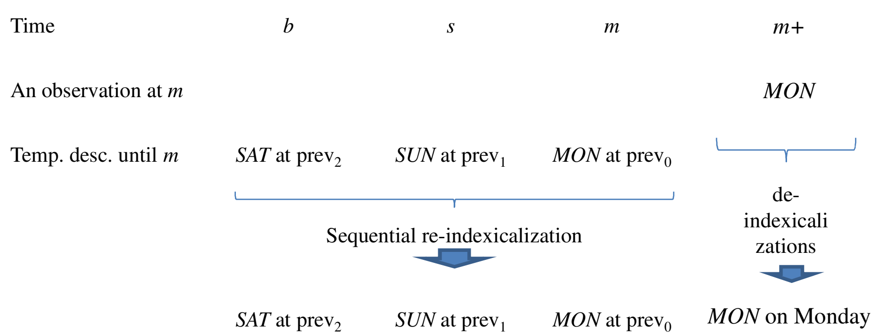

Figure 10: Reindexicalization and Deindexicalization 1. The sequential de priori information (MON at prev$_1$)&(SUN at prev$_2$)&(SAT at prev$_3$) is re-indexicalized to (MON at prev$_0$)&(SUN at prev$_1$)&(SAT at prev$_2$) and the

We can simplify (13) by using these facts: Jane was sure at m that *MON* is true on Monday, it was Sunday at the last epistemic moment (=*s*), and it was Saturday two epistemic moments ago (=*b*). At *m*, she believed *MON* to the degree of 1/2. Hence,

(14) $C_{m+}(R) = C_m(R$ on Monday/$MON) = 2C_m((R$ on Monday)&$MON)$.

Let R^*=(R on Monday)&MON; thus, $C_{m+}(R)=2C_m(R^*)$. To use this equation, we need to find her credence at m in R^*.

Focus on Jane's credal transition from s to m: She is not sure at m whether it is Monday or Tuesday, but she remembers that it was Sunday at the previous moment and that it was Saturday two moments ago. By GSJC,

(15) $C_m(R^*)$=

$C_s(R^*$ on Monday/(P on Monday)&(SUN at prev_0)&(SAT at prev_1))

$C_m((P\&MON)\&(SUN$ at $\text{prev}_1)\&(SAT$ at $\text{prev}_2)$)+

$C_s(R^*$ on Tuesday/(P on Tuesday)&(SUN at prev_0)&(SAT at prev_1))

$C_m((P\&TUE)\&(SUN$ at $\text{prev}_1)\&(SAT$ at $\text{prev}_2))$.

For [P on Monday] is the de-indexicalization of P under MON, and [P on Tuesday] is that of P under MON, and (SUN at prev_0)&(SAT at prev_1) is the re-indexicalization of (SUN at prev_1)&(SAT at prev_2) for the previous epistemic moment, as in Figure 11. Also, observe these facts: (i) [R^* on Monday] is equivalent to [R on Monday].[45] (ii) [R^* on Tuesday] is impossible.[46] (iii) Jane was sure at s of [SUN at prev_0]&[SAT at prev_1]. (iv) Jane is sure at m of [SUN at prev_1]&[SAT at prev_2]. Hence,

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[45] For [((R on Monday)&MON) on Monday]=[((R on Monday) on Monday)&(MON on Monday)]=(R on Monday).

[46] For [((R on Monday)&MON) on Tuesday] entails (MON on Tuesday), which is a contradiction.

(16) $C_m(R^*)=C_s(R \text{ on Monday}/P \text{ on Monday})C_m(P\&MON)=0.4.$

From (14) and (16), it follows that her credence at $m+$ in R is 0.8. This is an intuitive result, as her credence at s in raining on Monday was 0.8, conditional on precipitation on Monday. She later learns that there is a form of precipitation today and that today is Monday. Hence, it follows that there is a form of precipitation on Monday after all. Given these facts, it is natural that her credence at $m+$ ($\in$ Monday) is 0.8 in R.

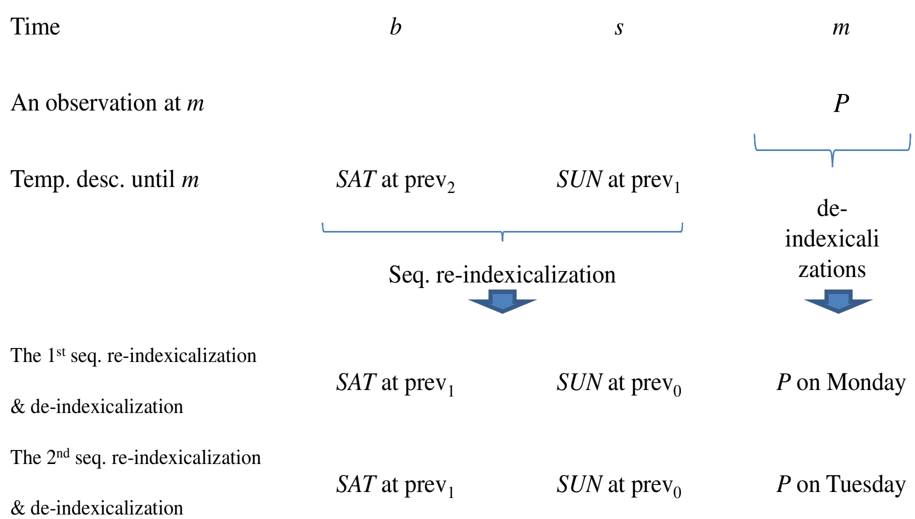

Figure 11: Reindexicalization and Deindexicalization 2. The observation P is de-indexicalized to (P on Monday) and (P on Tuesday), and sequential *de priori* information (SUN at prev_1)&(SAT at prev_2) is re-indexicalized to (SUN at prev_0)&(SAT at prev_1).

Finally, I want to formulate GSJC and GSSC in more succinct forms: Let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be an agent B's general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k_o \text{ at prev}_{m-k})$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}(\, W^{m+k}_o$ at

$\text{prev}_{m+k\text{-}1}$). Given this partition, I introduce these definitions: (i) $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is *probabilistically optimal* for X iff for any $o\in O$, $C_{n+m}(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)$ is conditioned upon a well-specified temporal description, where $\mathcal{D}_o=\&_{1\le k\le m}(E^k{}_o \text{ in } v^k{}_o)$ and $\mathcal{R}_o=\&_{1\le k\le n+1}(W^k{}_o \text{ at } \text{prev}_{k\text{-}1})$. Also, remember the definition of logical optimality in the last chapter. Given these two definitions, (ii) $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is *optimal* for X iff $\{\mathcal{E}_o\&\mathcal{V}_o\}_{o\in O}$ is logically optimal for X and $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is probabilistically optimal for X. Then,

(GSJC) $C_{n+m}(X)=\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$ if $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is optimal for X,

where $\mathcal{D}_o=\&_{1\le k\le m}(E^k{}_o \text{ in } v^k{}_o)$ and $\mathcal{R}_o=\&_{1\le k\le n+1}(W^k{}_o \text{ at } \text{prev}_{k\text{-}1})$. Next, let $\{\mathcal{E}\&\mathcal{V}\&\mathcal{W}\}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}=\&_{1\le k\le m}(E^k \text{ at } \text{prev}_{m\text{-}k})$, $\mathcal{V}=\&_{1\le k\le m}(V^k \text{ at } \text{prev}_{m\text{-}k})$ and $\mathcal{W}=\&_{1\le k\le n+1}(W^{m+k} \text{ at } \text{prev}_{m+k\text{-}1})$. Then,

(GSSC) $C_{n+m}(X)=C_n(X \text{ in } v^m/\mathcal{D}\&\mathcal{R})$ if $\{\mathcal{E}\&\mathcal{V}\&\mathcal{W}\}$ is optimal for X,

where $\mathcal{D}=\&_{1\le k\le m}(E^k \text{ in } v^k)$ and $\mathcal{R}=\&_{1\le k\le n+1}(W^k \text{ at } \text{prev}_{k\text{-}1})$. Obviously, these reformulations are equivalent to the original.

So far, I have presented GSJC and GSSC, illustrated how they work with an example, and provided shorter formulations for them. The next step is to defend the new updating principles with a new variant of the Conditional Expert Principle.

D. Temporal Conditional Multiple Expert Principle

In the last chapter, I introduced the Temporal Conditional Expert Principle. That principle described an epistemic relation between *two* agents located at different times. As such, it takes only *their* times and observations into consideration. In this section, I will present a new principle, which is similar to TCE but assumes additional agents contributing to the epistemic cooperation.

First, consider two agents B_{n+m}, located at time t_{n+m}, and B_n, located at time t_n, where $t_{n+m}>t_n$; for convenience, I will call B_{n+m} "the client" and B_n "the expert." As the names indicate, the client wants to set her credences at t_{n+m} by consulting B_n's credal opinion at t_n. (I will often omit "at t_{n+m}" and "at t_n.") Here, I suppose that the expert's credal judgment is not dependent upon any other agent's data or judgment. See Figure 12. Call this type of situation "a two agent situation." In this situation, what is the rational way for the client to set her credences by checking the expert's?

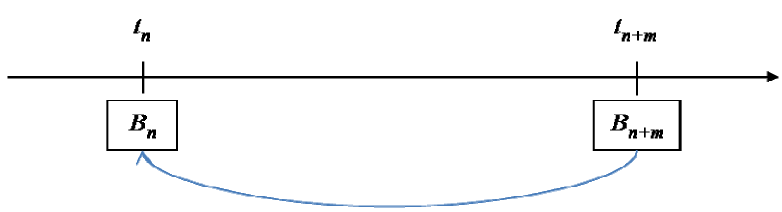

Figure 12: Judgmental Dependence. The arrow line indicates the judgmental dependence. There is no informational dependence on any other agents.

Earlier, I argued for the following answer: For any tensed proposition X,

(TCE) $C_{n+m}(X/\mathcal{E}\&\mathcal{V}\ \&pr_n(X'/\mathcal{E}')=r)=r$ if the left-hand side has a defined value and the following conditions are satisfied:

a) C_{n+m} is the client's credence function at t_{n+m}, and pr_n is the expert's credence function at t_n.

b) All information had at t_{n+m} by the client is accessible to the expert at t_n, possibly except $\mathcal{E}$, and the expert perhaps does not know at t_n whether $\mathcal{E}'$ is true.

c) X' and $\mathcal{E}'$ are tensed propositions such that the client knows at t_{n+m} that if $\mathcal{V}$ is true, [X is presently true iff X' was true at t_n] and [$\mathcal{E}$ is presently true iff $\mathcal{E}'$ was true at t_n].

I briefly repeat the rationale that I provided for this principle: By (a), it seems rational that C_{n+m} restricts C_n given the agent's intention to consult the expert's opinion in order to set her *de nunc* credences. By (b), it will however be irrational for the client to set her credence in X to be simply the same as the credence that the expert assigned to X. By (c), the client will think that if $\mathcal{V}$ is true, it is best to assign r to X conditional on $\mathcal{E}$ provided that *Ex*'s credence in X' given $\mathcal{E}'$ is r; for, she knows that if $\mathcal{V}$ is true, [X is presently true iff X' was true when the expert had the consulted credal opinion] and [$\mathcal{E}$ is presently true iff $\mathcal{E}'$ was true when the expert had the consulted credal opinion].

I still believe that TCE makes sense when there are no agents other than the client and the expert to take into consideration. However, think about the following situation: Consider $n+m+1$ agents, B_{n+m}, …, B_n, …, B_0, located at different moments, t_{n+m}, …, t_n, …, t_0, where $t_{n+m}>\ldots>t_0$; for convenience, we will call B_{n+m} "the client," B_{n+m}, …, B_{n+1} "the direct data providers," B_n "the expert," and B_n, …, B_0 "the indirect

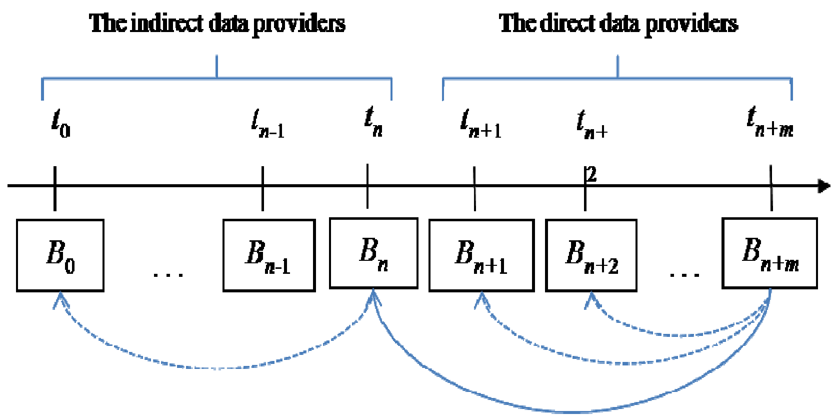

Figure 13: Direct and Indirect Data Providers. Here, B_{n+m} is the client and B_n is the expert. The dashed arrow line indicates the informational dependence, while the solid arrow line—from B_{n+m} to B_n—indicates the judgmental dependence.

data providers." We suppose that the client intends to judge the probability of X with the help of the expert, given the data observed by the direct data providers. Also, the expert's judgment, consulted by the client, is based upon the data observed by the indirect data providers. See Figure 13. Call this situation "a multiple agent situation."[47] In this situation, what is the rational way for the client to set her present credence?

As an answer, I suggest a principle that I call "the Temporal Conditional Multiple Expert Principle": Let $\mathcal{E}$ be a tensed proposition specifying the observations that

[47] In their paper (forthcoming), Dietrich and List discuss the topic of how one can *aggregate* the opinions of multiple agents in a rational way. I believe that such a theory will be useful also for clarifying how a single person can aggregate the opinions of her multiple selves in the past in a rational way. Another important topic, which has been left unstudied as far as I know, is that of how an individual is supposed to aggregate the *de se* opinions of multiple agents.

have been made by the direct data providers, let $\mathcal{V}$ be a tensed proposition about in what times the direct data providers are located, and let $\mathcal{W}$ be a tensed proposition about in what times the indirect data providers are located. Then, for any tensed proposition X,

(TCME) $C_{n+m}(X/\mathcal{E}\&\mathcal{V}\&\mathcal{W}\&pr_n(X'/\mathcal{E}'\&\mathcal{W}')=r)=r$, if the left-hand side has a defined value and the following conditions are satisfied:

(d) C_{n+m} is the client's credence function at t_{n+m} and pr_n is the expert's credence function at t_n.

(e) All information had at t_{n+m} by the client is accessible to the expert at t_n possibly except $\mathcal{E}$, and so the expert perhaps does not know at t_n whether $\mathcal{E}'$ is true.

(f) X' and $\mathcal{E}'$ are tensed propositions such that the client knows at t_{n+m} that if $\mathcal{V}$ is true, [X is presently true iff X' was true at t_n] and [$\mathcal{E}$ is presently true iff $\mathcal{E}'$ was true at t_n].

(g) $\mathcal{W}'$ is a tensed proposition such that B knows at t_{n+m} that [$\mathcal{W}$ is presently true iff $\mathcal{W}'$ was true at t_n].

Here, the main difference is (d): According to TCE, the client does not have to take the temporal locations of the agents providing data to the expert. According to TCME, the client needs to take those agents' temporal locations into consideration; conditional on the assumption that the indirect data providers were located at the times specified by $\mathcal{W}$, the client needs to consult the expert's credence in X' not only conditioned upon $\mathcal{E}'$, but also upon $\boldsymbol{\mathcal{W}}'$, where $\boldsymbol{\mathcal{W}}'$ specifies the same temporal locations of the indirect data providers as specified by $\boldsymbol{\mathcal{W}}$.

Why this difference? In a multiple agent situation, the client will be aware that the expert's credal opinion was made by depending upon the data from the indirect data providers. In order for the expert to correctly interpret those data, he will need the information about *when* those data were observed; in other words, he will need the information about the indirect data providers' temporal locations.

To appreciate this point, let's consider an analogous example: **Example 4**. Four meteorologists are flying in balloons in the New England sky. Let's call them "B_3," "B_2,""B_1," and "B_0," in the order of spatial proximity to B_3. Suppose that B_3 is judging the probability of rain in her region with the help of the other meteorologists. I assume that each B_k is equipped with a walkie-talkie but does not communicate with the other meteorologists unless B_k is contacted by a $B_{i>k}$ or needs the data or judgment of some $B_{i<k}$. (Hence, the information flows from B_0 to B_3 but not in the other direction.) In this situation, what will be the best strategy for B_3 to make a credal judgment about rain in her region?

One good strategy would be for her to "delegate" some required tasks to, say, B_1, so that while B_3 gathers *data* from B_3 and B_2, she depends upon the *judgment* of B_1. (Note that this is not to disregard the observations made by B_1 and B_0 because, if rational, B_1 will judge by taking their observations into consideration.) Let's call B_3 "the client," B_3 and B_2 "the direct data providers," B_1 "the expert," and B_1 and B_0 "the indirect data providers."

First, the client will need the information about the direct data providers' spatial locations in order to correctly judge the probability of rain using their data. This is because without knowing where the observed events are occurring, it will be difficult to correctly judge the relevance of the observations made by the direct data providers to the possibility of rain in her region.[48] For example, suppose that $\mathcal{E}$ represents the data observed by the direct data providers, where $\mathcal{E}$=(5°C temperature is being observed by B_3)&(thick clouds are being observed by B_2). To correctly interpret $\mathcal{E}$, the client (=B_3) will need information about where the direct data providers observed the conjuncts of $\mathcal{E}$, such as $\mathcal{L}$=(B_3 is located in Amherst)&(B_2 is located in Pelham).

Second, the client also will need the spatial locations of the indirect data providers in order to rationally utilize the expert's credal judgment. Intuitively, given $\mathcal{E}$&$\mathcal{L}$, the client will consult the expert's probability of rain in Amherst conditioned upon $\mathcal{E}'$, where $\mathcal{E}'$=(5°C temperature is being observed in Amherst)& (thick clouds are being observed in Pelham). Now, if the expert is rational, he will make this credal judgment on the basis of the data from the indirect data providers. Consequently, *the expert will need the spatial locations of the indirect data providers to correctly interpret the data from them, just as the client needs the spatial locations of the direct data providers to correctly interpret their data.*

To see this point clearly, suppose that the expert has the information that ($\mathcal{F}$) a strong wind is observed by B_1, and the wind is observed to be blowing from east to west by B_0. Then, compare two possible spatial locations of B_1 and B_0:

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[48] Also, she will need to know the region in which she is located to know the region for which she is judging the probability of rain.

($\mathcal{M}$) B_1 is located in Belchertown, and B_0 is located in Ware.

($\mathcal{M}$*) B_1 is located in Newton, and B_0 is located in Cambridge.

From the point of view in Amherst, a strong east wind in nearby eastern areas such as Belchertown and Ware will elevate the probability of rain in Amherst given $\mathcal{E}'$, but the same wind in distant lands like Newton and Cambridge will not be particularly relevant to the possibility of rain in Amherst on the condition of $\mathcal{E}'$.[49] Hence,

(17) pr_n(raining in Amherst/$\mathcal{E}'$&$\mathcal{M}$)$\neq pr_n$(raining in Amherst/$\mathcal{E}'$&$\mathcal{M}$*),

where pr_n is the expert's credence function. Now, assume that the expert was almost sure of $\mathcal{M}$*, so that

(18) pr_n(raining in Amherst/$\mathcal{E}'$)$\approx pr_n$(raining in Amherst/$\mathcal{E}'$&$\mathcal{M}$*),

but also that the client is quite sure of $\mathcal{M}$. In that case, it will be irrational that

(19) C_{n+m}(raining /$\mathcal{E}$&$\mathcal{L}$ &pr_n(raining in Amherst/$\mathcal{E}'$)=r)=r if defined,

[49] For the relevant geographical data, see the entry of "Massachusetts" in http://www.wikipedia.org.

where C_{n+m} is the client's credence function. For the expert's above conditional credence was, from the client's point of view, a judgment largely based upon *wrong* information about the indirect data providers' spatial locations.

This problem is due to the potential difference between the client's and the expert's opinions about the indirect data providers' spatial locations. One way to *bracket out* this difference is to consult the expert's credence *conditioned upon* the (from the client's point of view) correct information about the spatial locations of the indirect data providers. This suggests the following relation between the client's and the expert's credal opinions: For any real number r,

(20) $C_{n+m}(\text{raining}/\mathcal{E}\&\mathcal{L}\&\mathcal{M}\&pr_n(\text{raining in Amherst}/\mathcal{E}'\&\mathcal{M})=r)=r$ if defined.

It is not difficult to apply the same idea to a case in which the agents are located at different *times*; consider **example 5**. Four meteorologists have made observations about the weather of Amherst on different days. Call the meteorologists "B_3," "B_2,""B_1," and "B_0," in the reverse order of time. We suppose that B_3, "the client," is making a credal judgment about whether it will rain today with the help of the other three agents. In particular, she depends upon the data from B_3, B_2, "the direct data providers," and the judgment of B_1, "the expert." It is clear that she also comes to indirectly rely upon the data from B_1, B_0, "the indirect data providers." In this case, what will be the correct way for the client to make her credal judgment about whether it will rain in Amherst today?

First, she will need to know the temporal locations of the direct data providers in order to correctly interpret the data from them. Let $\mathcal{E}$=(5°C temperature is being observed by B_3)&(thick clouds were observed by B_2), and $\mathcal{V}$=(B_3 is located in Saturday)&(B_2 is located in Sunday). Given $\mathcal{V}$, the client can process $\mathcal{E}$ into the equivalent data $\mathcal{E}'$=(the 5°C temperature is being observed on Sunday)&(thick clouds were observed on Sunday).

Second, she will need to know the temporal locations of the indirect data providers in order to correctly utilize the expert's credal judgment. For the expert's credal opinion must have been produced on the bases of their data. Suppose that the expert had the information that ($\mathcal{F}$) B_0 had met somebody telling him that for the next three days, once thick clouds have formed, they will not go away quickly, and B_1 was told that the guy she met yesterday was a very good expert about cloud forming. Consider these temporal locations of the indirect data providers:

($\mathcal{W}$) B_1 was located on Saturday, and B_0 had been located on Friday.

($\mathcal{W}$*) B_1 was located on Thursday, and B_0 had been located on Wednesday.

Clearly,

$$(21) \qquad pr_n(\text{raining on Monday}/\mathcal{E}'\&\mathcal{W}) \neq pr_n(\text{raining on Monday}/\mathcal{E}'\&\mathcal{W}^*),$$

where pr_n is the expert's credence function. Assuming that the client is almost sure of $\mathcal{W}$, but the expert is almost sure of $\mathcal{W}$*, it will be irrational that

(22) $C_{n+m}(\text{raining } /\mathcal{E}\&\mathcal{V}\&pr_n(\text{raining on Monday}/\mathcal{E}')=r)=r$ if defined,

where C_{n+m} is the client's credence function. For the expert's above conditional credence is largely based on, from the client's point of view, *wrong* information about the indirect data providers' temporal locations, which was essential to judging correctly $\mathcal{E}'$'s relevance to whether or not it will rain on Monday. Rather, the rational way that the client would utilize the expert's opinion is the following:

(23) $C_{n+m}(\text{raining } /\mathcal{E}\&\mathcal{V}\&\mathcal{W}\&pr_n(\text{raining on Monday}/\mathcal{E}'\&\mathcal{W})=r)=r$ if defined.

Until now, we have discussed cases in which the client has a means of specifying the indirect data providers' locations in *non-indexical* ways. However, I do not think the lesson we have learned from these examples depends upon the existence of such a non-indexical method of specifying the indirect data providers' locations. So let $\mathcal{W}$ and $\mathcal{W}'$ be the tensed propositions specifying the indirect data providers' temporal locations such that from the client's point of view, (*) $\mathcal{W}$ is presently true iff $\mathcal{W}'$ was true at the expert's time. Then, it will be the case that

(24) $C_{n+m}(X/\mathcal{E}\&\mathcal{V}\&\mathcal{W}\&pr_n(X'/\mathcal{E}'\&\mathcal{W}')=r)=r$ if defined,

where X' and $\mathcal{E}'$ satisfy TCME's provisos (a)-(c) with respect to X, $\mathcal{E}$, and $\mathcal{V}$. For $\mathcal{W}$ tells the client that $\mathcal{W}'$ was [the tensed proposition specifying the indirect data providers' temporal locations] that was true at the expert's time. By introducing (*) as the fourth proviso (d), we acquire the general principle TCME.

In this section, I have argued that we need to modify TCE in order to capture the rational method for an agent to defer to the credal judgment of another agent located at an earlier moment, where both agents are provided relevant data.

E. A Defense of GSJC

In this section, I will present a new principle, "General Shifted Sequential Rigidity" (hereafter: GSR), and defend it by making use of TCME. After defending it, I will point out that GSR entails GSJC. Since GSSC is a special case of GSJC, the two principles presented in the last section will have been defended.

I start by presenting GSR: Let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be B's general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k_o \text{ at prev}_{m-k})$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}(W^k_o \text{ at prev}_{m+k-1})$. Next, suppose that $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is optimal for X. Then,

$$\text{(GSR)} \qquad C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(X \text{ in } v^m_o/\mathcal{D}_o\&\mathcal{R}_o),$$

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the m epistemic moments earlier time. (In other words,

$\mathscr{D}_o$=$\&_{1\leq k\leq m}(E^k{}_o$ in $v^k{}_o$) and $\mathscr{R}_o$=$\&_{1\leq k\leq n+1}(W^k{}_o$ at prev$_{k-1}$).) It is not difficult to see that GSR entails GSJC.[50]

How do we defend GSR? We defend it by using TCME. First, let B_{n+m}, …, B_0 be the agent B's selves at t_{n+m}, …, t_0, where t_{n+m}>…>t_0. We suppose that B_{n+m} sets her credence in X with the help of B_{n+m-1}, …, B_0. One way to do so will be to collect the data from B_{n+m}, …, B_{n+1} and delegate the job of making a suitable conditional credal judgment to her past self, say, B_n. Accordingly, let's call B_{n+m} "the client" and B_n "the expert."

One important question is whether this is a two agent situation or a multiple agent situation. Clearly, this is a case of the latter. Not only does the client depend upon B_{n+m}, …, B_{n+1} to acquire extra data, but the expert must also depend upon B_n, …, B_0 to make the suitable conditional credal judgment. Accordingly, let's call B_{n+m}, …, B_{n+1} "the direct data providers" and B_n, …, B_0 "the indirect data providers."

Once put in this way, it is plausible that TCME applies to this case, since I have previously argued that in a multiple agent case, TCME, not TCE, is the principle describing the epistemic relation between the client and the expert. Then, we can derive the principle that I call the "General Shifted Backward Reflection Principle" from TCME: Let X be any tensed proposition and $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be B's general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o$=$\&_{1\leq k\leq m}(E^k{}_o$ at prev$_{m-k}$), $\mathcal{V}_o$=$\&_{1\leq k\leq m}(V^k{}_o$ at prev$_{m-k}$), and $\mathcal{W}_o$=$\&_{1\leq k\leq n+1}(W^k{}_o$ at prev$_{m+k-1}$). Next, let $\mathscr{D}_o$ be the sequential de-indexicalization of $\mathcal{E}_o$

[50] Suppose GSR. So $C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$=$C_n(X$ in $v^m{}_o/\mathscr{D}_o\&\mathscr{R}_o)$ for any $o\in O$. Since $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is a general time-observation partition from t_n to t_{n+m}, (i) $\sum_{o\in O}C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$=1 and (ii) $(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$>0 for each $o\in O$. Thus, $C_{n+m}(X)$=$\sum_{o\in O}C_{n+m}(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$=$\sum_{o\in O}C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$= (by supposition) $\sum_{o\in O}C_n(X$ in $v^m{}_o/\mathscr{D}_o\&\mathscr{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$. Done.

under $\mathcal{V}_o$ (i.e., $\mathcal{D}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)$), and let $\mathcal{R}_o$ be the sequential re-indexicalization of $\mathcal{W}_o$ for the m epistemic moments earlier time (i.e., $\mathcal{W}_o = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1})$). Assume that $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is optimal for X. Then,

(GSBR) $\quad C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\&C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)=r)=r$ if defined.

For (a) C_{n+m} is the client's credence function, and C_n is the expert's, (b) $\mathcal{E}_o$ specifies the data accessible to the client but perhaps not to the expert, (c) the client knows (at t_{n+m}) that X is presently true iff [X in $v^m{}_o$] was true m epistemic moments ago, and $\mathcal{E}_o$ is presently true iff $\mathcal{D}_o$ was true m epistemic moments ago, and (d) the client also knows (at t_{n+m}) that $\mathcal{W}_o$ is presently true iff $\mathcal{R}_o$ was true m epistemic moments ago. Since the elements composing GSBR satisfy the provisos of TCME, GSBR is a special case of TCME. GSR is derivable from GSBR under the presupposition that B always remembers her past credence function with perfect confidence and correctness.[51]

Remember that GSBR entails GSR and GSR entails GSJC. Therefore, we have good reason to accept GSJC.

F. Too Far Past Does Not Matter

The earlier formulation of GSJC has a practical problem: In most credal transitions, we do not worry about what time it was at t if t is a moment sufficiently far past. Even in such a case, GSJC asks us to take such a matter into consideration. In this section, I will

[51] Let $r=C_n(X \text{ in } v_j/\mathcal{D}_o\&\mathcal{R}_o)$. By the presupposition of perfect memory, $C_{n+m}(C_n(X \text{ in } v_j/\mathcal{D}_o\&\mathcal{R}_o))=r)=1$. Thus, $C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\&C_n(X \text{ in } v_j/\mathcal{D}_o\&\mathcal{R}_o)=r)=$(by GSBR)$r=C_n(X \text{ in } v_j/\mathcal{D}_o\&\mathcal{R}_o)$. Done.

provide another formulation of GSJC, which is (i) relatively free from this problem and (ii) still equivalent to the original formulation of GSJC.

First, I review GSJC: Let $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ be an agent B's general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$, $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o = \&_{1\leq k\leq n+1}(V^k{}_o \text{ at prev}_{m+k-1})$. Then,

(GSJC) $C_{n+m}(X) = \Sigma_{o\in O} C_n(X \text{ in } v^m{}_o / \mathcal{D}_o \& \mathcal{R}_o) C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)$ if $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ is optimal for X,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ (i.e., $\mathcal{D}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)$) and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the m epistemic moments earlier time (i.e., $\mathcal{R}_o = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1})$).

We can expect a complaint: If I update in accordance with GSJC, then I have to presently assign to X the weighted average of [X in $v^m{}_o$] given $\mathcal{D}_o \& \mathcal{R}_o$ (with the weights coming from…), where $\mathcal{R}_o$ specifies what times it had been at *all my epistemic moments until that from which I am updating*. These epistemic moments even include the moment of my first observation. This appears to be absurdly demanding. For example, why must I take my birth time into consideration, in judging the probability of rain today?

This is a fair complaint. There should be a way in which one can rationally judge the probability of rain today without worrying about when she was born, when she fell in love for the first time, etc. So, I reformulate GSJC into a principle that does not mention epistemic moments that are too far in the past to be relevant. First, I suggest a modified definition of general time-observation partition. The core idea is that first we can remove from the given general time-observation partition its elements ascribing times to epistemic moments too far in the past to be relevant, and second, we can formulate the updating principle that works with the remaining partition.

Here is the first step: Again, let $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ be an agent B's general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o = \mathop{\&}_{1\leq k\leq m}(E^k_o \text{ at prev}_{m-k})$, $\mathcal{V}_o = \mathop{\&}_{1\leq k\leq m}(V^k_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o = \mathop{\&}_{1\leq k\leq n+1}(W^k_o \text{ at prev}_{m+k-1})$. Consider some $i\in\{1,\ldots,n+1\}$. Given $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ and i, let $\{\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p\in P}$ be the maximal partition such that for each $p\in P$, there exists $o\in O$ such that $\mathcal{E}_p=\mathcal{E}_o$, $\mathcal{V}_p=\mathcal{V}_o$, and $\mathcal{W}_p = \mathop{\&}_{1\leq k\leq n-i}(W^k_o \text{ at prev}_{m+k-1})$.[52] Given these partitions, I will say that $\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p$ is the *abbreviation* of $\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o$. It follows that for any $o\in O$, there exists $p\in P$ such that $\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o = (\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p \& \mathcal{W}^*_p)$, where $\mathcal{W}^*_p = \mathop{\&}_{n-i+1\leq k\leq n+1}(W^{m+k}_o \text{ at prev}_{m+k-1})$. In such a case, I will say that $\mathcal{W}^*_p$ is a *complement* of $\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p$ for $\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o$. The point is that $\{\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p\in P}$ is the same as $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ except that each $\mathcal{W}_p$ may include only a fragment of corresponding $\mathcal{W}_o$. In such a case, I will say that $\{\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p\in P}$ is (B's) *general time-observation partition* from t_n to t_{n+m} over $[t_i, t_{n+m}]$.[53] Also, I will say that $\{\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p\in P}$

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[52] Here, I suppose that if "$1\leq k\leq n-i$" is not satisfied by any number k, then $\mathop{\&}_{1\leq k\leq n-i}(W^k_o \text{ at prev}_{m+k-1})$ is vacuously true. For example, if $i=n+1$, $\&_{1\leq k\leq n-i}(W^k_o \text{ at prev}_{m+k-1}) = \&_{1\leq k\leq -1}(W^k_o \text{ at prev}_{m+k-1}) = T$, where T is any tautology.

[53] Note: If $i=n+1$, then $\mathcal{W}_p$ is vacuous (see the previous footnote) and so $\{\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p\in P} = \{\mathcal{E}_p \& \mathcal{V}_p\}_{p\in P} = \{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$.

is *sufficiently inclusive for* a tensed proposition X iff for each $o \in O$ and $p \in P$, if $\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p$ is an abbreviation of $\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o$ and $\mathcal{W}^*_p$ is the complement, then $C_n(X$ in $v^m{}_p / \mathcal{D}_p \& \mathcal{R}_p) = C_n(X$ in $v^m{}_p / \mathcal{D}_p \& \mathcal{R}_p \& \mathcal{R}^*_p)$, where $\mathcal{D}_p$ is the sequential de-indexicalization of $\mathcal{E}_p$ under $\mathcal{V}_p$ and $\mathcal{R}_p$ and $\mathcal{R}^*_p$ are the sequential re-indexicalizations of $\mathcal{W}_p$ and $\mathcal{W}^*_p$ for the m epistemic moments earlier time.

Here is the second step: Let $\{\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p \in P}$ be a general time-observation partition from t_n to t_{n+m} over $[t_i, t_{n+m}]$. Then,

(GSJC$^-$) $C_{n+m}(X) = \Sigma_{p \in P} C_n(X$ in $v^m{}_p / \mathcal{D}_p \& \mathcal{R}_p) C_{n+m}(\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p)$ if $\{\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p \in P}$ is optimal and sufficiently inclusive for X,

where $\mathcal{D}_p$ is the sequential de-indexicalization of $\mathcal{E}_p$ under $\mathcal{V}_p$, and $\mathcal{R}_p$ is the sequential re-indexicalization of $\mathcal{W}_p$ for the m epistemic moments earlier time. Of course, it is possible to formulate the corresponding variant of GSSC: Let $\{\mathcal{E} \& \mathcal{V} \& \mathcal{W}\}$ be a general time-observation partition from t_n to t_{n+m} over $[t_i, t_{n+m}]$. Then,

(GSSC$^-$) $C_{n+m}(X) = C_n(X$ in $v^m{}_p / \mathcal{D} \& \mathcal{R})$ if $\{\mathcal{E} \& \mathcal{V} \& \mathcal{W}\}$ is optimal and sufficiently inclusive for X,

where $\mathcal{D}$ is the sequential de-indexicalization of $\mathcal{E}$ under $\mathcal{V}$, and $\mathcal{R}$ is the sequential re-indexicalization of $\mathcal{W}$ for the m epistemic moments earlier time. It is provable that GSJC- is equivalent to GSJC and that GSSC- is equivalent to GSSC. (See APPENDIX A.)

In GSJC$^-$ and GSSC$^-$, epistemic moments that are too far past are not mentioned as long as "What times were it in those moments?" is an irrelevant question to how probable the target tensed proposition is now. Hence, we now have more practical variants of GSJC and GSSC.

G. <u>Application to the SB Problem</u>

So far, I have developed a more general rule for *de nunc* updating than SSJC. This generalization was initially motivated by the fact that SSJC does not apply correctly to SB's credal transition from *m* to *m*+. Hence, it will be interesting to see how well our new rule, GSJC, will do with respect to the same credal transition.

For an easier discussion, I will first apply GSJC to a variant of the SB problem. The lessons from the variant problem will help us to understand how to apply GSJC to the original problem. Think about this version of the SB problem: **SB problem 3**. SB is *born* on Sunday, knowing that it is Sunday. She also knows the following events will happen during the next three days: Immediately after her birth, a group of evil experimenters put her to sleep. Next, they toss a fair coin. Case 1: (*H*) The coin lands heads. Then, they wake her up once on Monday. Case 2: (*T*) The coin lands tails. In this case, the experimenters wake her up twice, the first time on Monday and the second time on Tuesday. Between the two awakenings, they inject her with a drug that erases her memory of the first awakening. In either case, SB is told that it is Monday one minute after she wakes up on Monday. Here is the question: What is her credence in *H* when she is told that it is Monday?

Let *s* be the moment of her birth on Sunday, *m* be the moment of her wakeup on Monday, and *m*+ be the moment of being told that it is Monday. There are two ways in which we can answer the above question by using GSJC. First, we can use GSSC for her credal transition from m to $m+$:

(25) $C_{m+}(H)=C_m(H$ on Monday/(MON on Monday)&(MON at prev$_0$)&(SUN at prev$_1$)).

Since H is a genuine proposition, [H on Monday] is equivalent to H. It is a tautology that MON is true on Monday. Finally, when she wakes up on Monday, she remembers that it was previously Sunday. Hence,

(26) $C_{m+}(H)=C_m(H\,/MON$ at prev$_0)=C_m(H/MON)=C_m(H\&MON)/C_m(MON)$.

To find the last value, we use GSJC for her credal transition from s to m. At m, SB is sure that she is observing W at that moment and that it was previously Sunday, but she does not know whether it is Monday or Tuesday. Thus, her general time-observation partition from s to m is {W&(MON at prev$_0$)&(SUN at prev$_1$), W&(TUE at prev$_0$)&(SUN at prev$_1$)}. Hence, the correct instance of GSJC is

(27) $C_m(H\&MON)=$

C_s((H&MON) on Monday/(W on Monday)&(SUN at prev$_0$))*

C_m(W&(MON at prev$_0$)&(SUN at prev$_1$))+

$C_s((H\&MON)$ on Tuesday/(W on Tuesday)&(SUN at prev_0))*

$C_m(W\&(TUE$ at $\text{prev}_0)\&(SUN$ at $\text{prev}_1))$.

Now observe the following facts: First, [($H\&MON$) on Monday] is clearly equivalent to H. Second, on Sunday, SB knew that it was Sunday, and she fully expected to wake up on Monday. Third, $H\&MON$ cannot be true on Tuesday. Thus, we can simplify (27) into

(28) $C_m(H\&MON)=C_s(H)*C_m(W\&(MON \text{ at } \text{prev}_0)\&(SUN \text{ at } \text{prev}_1))$.

On Sunday, SB believed to the degree of ½ that the coin would land heads. On Monday, she certainly knows that she is waking up with such and such a memory, and she remembers that it was previously Sunday. Therefore,

(29) $C_m(H\&MON)=1/2C_m(MON \text{ at } \text{prev}_0)=1/2C_m(MON)$.

It follows from (26) and (29) that her credence at $m+$ in H is ½. (Note that this result captures the intuition that I described in Section C.)

Second, GSSC provides the following instance for SB's credal transition from s to $m+$: When SB is told that it is Monday, she remembers that she previously experienced waking up with the memory of Sunday as the last memory, and she learns that it is Monday now. Hence, she is sure at $m+$ of (MON at prev_0)&(W at prev_1). Also, she is sure at that moment that it is Monday then and that it was Monday previously. Thus, she is sure at $m+$ of (MON at prev_0)&(MON at prev_1). Finally, she remembers that it was Sunday two epistemic moments ago. So, she is sure at m of (SUN at prev_2). Therefore,

(30) $C_{m+}(H)=C_s(H \text{ on Monday}/(MON \text{ on Monday})\&(W \text{ on Monday})\&(SUN \text{ at prev}_0))$,

because (MON on Monday)&(W on Monday) is the sequential de-indexicalization of (MON at prev_0)&(W at prev_1) under (MON at prev_0)&(MON at prev_1), and (SUN at prev_0) is the re-indexicalization of (SUN at prev_2) for the two epistemic moments earlier time. Of course, she fully expected on Sunday night that MON would be true on Monday and that she would wake up on Monday. Also, she knew on that night that it was Sunday. As a result,

(31) $C_{m+}(H)=C_s(H \text{ on Monday})=C_s(H)=1/2$.

Therefore, we arrive at the same conclusion whether we apply GSJC to SB's credal transition from s to m and then apply it to that from m to $m+$ step by step, or we apply it to her credal transition from s to $m+$ all at once.

This is an intuitive result, and the answer to the given question resembles the traditional Thirder view of the same question regarding the original SB problem. My next question is whether we can apply GSJC to the credal transitions in the *original* SB problem and acquire the same credence of hers at $m+$ in H.

Here, we are faced with a difficulty. In the original SB problem, it was not explicitly stated *when* SB made observations before the experiment began on Sunday night. Since that information is crucial for using GSJC, we cannot apply that rule to the original version of the SB problem.

What do we do? We can use GSJC^{-} instead. Here is the rough idea: Perhaps, there are many possibilities regarding when SB made observations before Sunday night. However, we can safely assume that those possibilities are irrelevant to how the coin lands on Monday. Under this assumption, we can apply GSJC^{-} and GSSC^{-} without worrying about the mentioned possibilities about what times it had been until Sunday night.

To precisify this idea, let $\{\mathcal{R}^{s}{}_{o}\}_{o\in O}$ be the partition whose members describe what days it had been at the epistemic moments before s, where $\mathcal{R}^{s}{}_{o}=(D^{1}{}_{o}$ at $\text{prev}_1)\&(D^{2}{}_{o}$ at $\text{prev}_2)\&(D^{3}{}_{o}$ at $\text{prev}_3)\&$ … . Let $\{\mathcal{R}^{m}{}_{o}\}_{o\in O}$ be a similar partition such that for each $o\in O$, $\mathcal{R}^{m}{}_{o}=(D^{1}{}_{o}$ at $\text{prev}_2)\&(D^{2}{}_{o}$ at $\text{prev}_3)\&(D^{3}{}_{o}$ at $\text{prev}_4)\&$ … . I make these assumptions:

(32) For any $o\in O$, $C_s(H/W$ on Monday$)=C_s(H/(W$ on Monday$)\&\mathcal{R}^{s}{}_{o})$ and $C_s(H/W$ on Tuesday$)=C_s(H/(W$ on Tuesday$)\&\mathcal{R}^{s}{}_{o})$.

(33) For any $o\in O$, $C_m(H/MON)=C_m(H/MON\&\mathcal{R}^{m}{}_{o})$.[54]

[54] I am assuming that $\mathcal{R}^{s}{}_{o}$ is well-specified *de priori* information with respect to <C_s, H, W on Monday> and <C_s, H, W on Tuesday>. Similarly, $\mathcal{R}^{m}{}_{o}$ is well-specified *de priori* information with respect to <C_m, H, MON>.

In words, from the point of view at s, what days it had been before now is irrelevant to H conditional on [W on Monday] and, from the point of view at m, what times it had been before the previous moment is irrelevant to H conditional on MON. These are highly plausible assumptions. After all, both $\mathcal{R}^s{}_o$ and $\mathcal{R}^m{}_o$ are tensed propositions describing what times it had been before Sunday night. Clearly, we have no reason to think that such a matter is relevant in judging the probability of the coin's landing heads.

Given these assumptions, I apply GSJC$^-$ to SB's credal transition from s to m: Consider partition $\{W\&MON\&(SUN \text{ at prev}_1)\&\mathcal{W}^m{}_o\}_{o\in O}\cup\{W\&TUE\&(SUN \text{ at prev}_1)\&\mathcal{W}^m{}_o\}_{o\in O}$, where $\mathcal{W}^m{}_o=(D^1{}_o \text{ at prev}_2)\&(D^2{}_o \text{ at prev}_3)\&(D^3{}_o \text{ at prev}_4)\&\ldots$. By definition, it is SB's general time-observation partition from s to m.[55] Also by definition, $\{W\&MON\&(SUN \text{ at prev}_1), W\&TUE\&(SUN \text{ at prev}_1)\}$ is a general time-observation partition from s to m over $[s,m]$. Furthermore, the doubleton is sufficiently inclusive.[56] Since it is the same partition I used for the credal transition from s to m in **SB problem 3**, (27) is also an instance of GSJC$^-$ for the credal transition from s to m in the original. By the same reasoning as in (27)-(29), $C_m(H\&MON)=1/2C_m(MON)$, i.e, (29) is also true in the original SB problem.

[55] From SB's point of view at m, W describes her present observation, (MON at prev_0) and (TUE at prev_0) describe the days that are possibly today, and (SUN at prev_1)$\&\mathcal{W}^m{}_o$ describe what times it had been until the previous moment.

[56] By (32), $C_s(H/W \text{ on Monday})=C_s(H/(W \text{ on Monday})\&\mathcal{R}^s{}_o)$ and $C_s(H/W \text{ on Tuesday})=C_s(H/(W \text{ on Tuesday})\&\mathcal{R}^s{}_o)$ for any $o\in O$. Since $C_S(SUN \text{ at prev}_0)=1$, $C_s(H/(W \text{ on Monday})\&(SUN \text{ at prev}_0))=C_s(H/(W \text{ on Monday})\&(SUN \text{ at prev}_0)\&\mathcal{R}^s{}_o)$ and $C_s(H/(W \text{ on Tuesday})\&(SUN \text{ at prev}_0))=C_s(H/(W \text{ on Tuesday})\&(SUN \text{ at prev}_0)\&\mathcal{R}^s{}_o)$ for any $o\in O$. Since (W on Monday) is the re-indexicalization of W under MON and $[(SUN \text{ at prev}_0)\&\mathcal{R}^s{}_o]$ is the sequential re-indexicalization of $((SUN \text{ at prev}_1)\&\mathcal{W}^m{}_o)$ for each $o\in O$, $\{W\&MON\&(SUN \text{ at prev}_1), W\&TUE\&(SUN \text{ at prev}_1)\}$ is sufficiently inclusive, by definition. Done.

Next, I apply $GSSC^-$ to SB's credal transition from m to $m+$. Consider this partition: $\{MON\&(MON \text{ at } prev_1)\&(SUN \text{ at } prev_2)\&\mathcal{W}^{m+}{}_o\}_{o\in O}$, where $\mathcal{W}^{m+}{}_o=(D^1{}_o \text{ at } prev_3)\&(D^2{}_o \text{ at } prev_4)\&(D^3{}_o \text{ at } prev_5)\& \ldots$. By definition, it is SB's general time-observation partition from m to $m+$.[57] Also, think about $\{MON\&(MON \text{ at } prev_1)\& (SUN \text{ at } prev_2)\}$. By definition, this singleton is her general time-observation partition from m to $m+$ over $[s,m+]$. Moreover, the second partition is sufficiently inclusive.[58] Therefore, (25) is an instance of $GSSC^-$ for the credal transition from m to $m+$. By the same reasoning as from (25) to (26) in **SB problem 3**, $C_{m+}(H)= C_m(H\&MON)/C_m(MON)$, i.e., (26) is also true in the original SB problem. From (26) and (29), it follows that $C_{m+}(H)=1/2$.

Finally, I apply $GSSC^-$ to SB's credal transition from s to $m+$. Consider this partition: $\{[(MON \text{ at } prev_0)\&(W \text{ at } prev_1)] \& [(MON \text{ at } prev_0)\&(MON \text{ at } prev_1)] \& \mathcal{W}^{m+}{}_o\}_{o\in O}$. By definition, it is SB's general time-observation partition from s to $m+$.[59] Plus, consider $\{[(MON \text{ at } prev_0)\&(W \text{ at } prev_1)] \& [(MON \text{ at } prev_0)\&(MON \text{ at } prev_1)]\}$. By definition, this singleton is a general time-observation partition from s to $m+$ over

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[57] From B's point of view at $m+$, MON describes her present observation and time, and $(MON \text{ at } prev_1)\&(SUN \text{ at } prev_2)\&\mathcal{W}^{m+}{}_o\}$ describes what times it had been until the previous moment.

[58] By (33), $C_m(H/MON)=C_m(H/MON\&\mathcal{R}^m{}_o)$. Since $C_m(SUN \text{ at } prev_1)=1$, $C_m(H/(MON \text{ on Monday})\&(MON \text{ at } prev_0)\&(SUN \text{ at } prev_1))=C_m(H/(MON \text{ on Monday})\&(MON \text{ at } prev_0)\&(SUN \text{ at } prev_1)\&\mathcal{R}^m{}_o)$. Since $(MON$ on Monday) is the de-indexicalization of MON under MON and $(MON \text{ at } prev_0)\&(SUN \text{ at } prev_1)\&\mathcal{R}^m{}_o$ is the sequential re-indexicalization of $(MON \text{ at } prev_1)\&(SUN \text{ at } prev_2)\&\mathcal{W}^{m+}{}_o$ for the one epistemic moment earlier time, $\{MON\&(MON \text{ at } prev_1)\& (SUN \text{ at } prev_2)\}$ is sufficiently inclusive.

[59] From B's point of view at $m+$, $[(MON \text{ at } prev_0)\&(W \text{ at } prev_1)]$ describes what observations she has made since the previous moment, $[(MON \text{ at } prev_0)\&(W \text{ at } prev_1)]$ describes what days it has been since the previous moment, and $\mathcal{W}^{m+}{}_o$ describes what days it has been since the two epistemic moments earlier time (from which she is updating).

[s,m+]. Also, the singleton is sufficiently inclusive.[60] Hence, (30) is also an instance of GSSC$^-$. Consequently, $C_{m+}(H)$=1/2.

In summary, we can apply GSJC$^-$, or equivalently GSJC, to SB's credal transitions from s to m and from m to m+ under some highly plausible assumptions, and that application leads to coherent results that comply with the popular Thirder view. This supports my suspicion that GSJC is the correct rule applicable to an agent's credal transition from when she did not know what time it was.

H. The Relation between GSJC and Other Rules

In this section, I discuss the relation between GSJC and other rules discussed in the earlier chapters. First, I will review RSSJC*, a restricted version of SSJC discussed in the last chapter. Second, I will present RSSJC, which is directly derivable from GSJC, and compare it with RSSJC*. Finally, I will discuss how the restricted versions of various rules in this dissertation are derived from GSJC.

First, remember this rule: Let $\{\mathcal{E}_o \& \mathcal{V}_o\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k_o$ at prev$_{m-k})$ and $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k_o$ at prev$_{m-k})$. Then, for any tensed proposition X,

[60] By (32), $C_s(H/W$ on Monday$)=C_s(H/(W$ on Monday$)\&\mathcal{R}^s_o)$ for any $o\in O$. Since $C_S((MON$ on Monday$)\&(SUN$ at prev$_0))=1$, $C_s(H/(MON$ on Monday$)\&(W$ on Monday$)\&(SUN$ at prev$_0))=C_s(H/(MON$ on Monday$)\&(W$ on Monday$)\&(SUN$ at prev$_0)\&\mathcal{R}^s_o)$. By definition, (MON on Monday)&(W on Monday) is the sequential de-indexicalization of (MON at prev$_0$)&(W at prev$_1$) under (MON at prev$_0$)&(W at prev$_1$), and (SUN at prev$_0$) and ((SUN at prev$_0$)&$\mathcal{R}^s_o$) are the sequential re-indexicalizations for the two epistemic moments earlier time. By definition, the above singleton is sufficiently inclusive. Done.

(RSSJC*) $C_{n+m}(X)= \sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o)$ if $\{\mathcal{E}_\text{o}\&\mathcal{V}_\text{o}\}_{o\in O}$ is logically optimal for X, and B was free from temporal ignorance at t_n,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ for each $o\in O$. Here, freedom from temporal ignorance at t_m means that at t_n B knew what time it was then. It does not imply that at t_n B knew anything about what time it was at the even earlier moments (which *we* know to be $t_{n\text{-}1}$, $t_{n\text{-}2}$, etc.). Since this is a restrictive rule, it is reasonable to expect that RGSJC* turns out to be a sub-principle of the new general rule, GSJC, i.e., all the instances of RGSJC* turn out to be those of GSJC.

Unfortunately, the proviso of RGSJC* is neither strong enough to guarantee that its instances are all derivable from GSJC nor weak enough to share all interesting instances of GSJC. To see this point, second, look at this rule: Let $\{\mathcal{E}_\text{o}\&\mathcal{V}_\text{o}\}_{o\in O}$ be B's sequential time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m\text{-}k})$ and $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m\text{-}k})$. Note that $\{\mathcal{E}_\text{o}\&\mathcal{V}_\text{o}\}_{o\in O}$ is also B's general time-observation partition from t_n to t_{n+m} over $[t_{n+1},t_{n+m}]$. Then, for any tensed proposition X,

(RSSJC) $C_{n+m}(X)= \sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o)$ if $\{\mathcal{E}_\text{o}\&\mathcal{V}_\text{o}\}_{o\in O}$ is logically optimal and sufficiently inclusive for X,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ for each $o\in O$. Clearly, this rule directly follows from GSJC$^-$, which is equivalent to GSJC.

To see the difference between RSSJC* and RSSJC, think about the logical relation between freedom from temporal ignorance and sufficient inclusion. Clearly, freedom from temporal ignorance is neither necessary nor sufficient for the given partition's being sufficiently inclusive. For $\{\mathcal{E}_o\&\mathcal{V}_o\}_{o\in O}$ is sufficiently inclusive for X exactly when (#) $C_n(X \text{ in } v^m{}_o/\mathcal{D}_o)=C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)$ for any $o\in O$, where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$, the complement of $\mathcal{E}_o\&\mathcal{V}_o$, for the m epistemic moments earlier time. To satisfy (#), either (i) B must have completely known at t_n what times it had been until then or (ii) her credence distribution at t_n must have been such that, for any $o\in O$, what times it had been is irrelevant to whether X will true in $v^m{}_o$ conditional on what observations she will make when. Even if B knew at t_n what time it was then, it is not sufficient to satisfy (i), and even if B did not know what time it was at t_n, (#) still can be satisfied if (ii) holds.

Given these facts, I find RSSJC to be more attractive as a restricted version of SSJC. Similar points hold for the other rules that I have discussed: Let $\{\mathcal{E}\&\mathcal{V}\}$ be B's general time-observation partition from t_n to t_{n+m} over $[t_{n+1},t_{n+m}]$. Then, for any tensed proposition X,

(RSSSC) $C_{n+m}(X)=C_n(X \text{ in } v^m/\mathcal{D})$ if $\{\mathcal{E}\&\mathcal{V}\}$ is logically optimal and sufficiently inclusive for X,

where $\mathcal{D}$ is the sequential de-indexicalization of $\mathcal{E}$ under $\mathcal{V}$. Let $\{E_o\&V_o\}_{o\in O}$ be B's time-observation partition from t_n to t_{n+1}. It is also her general time-observation partition from t_n to t_{n+1} over $[t_{n+1},t_{n+1}]$. Then, (RSJC) $C_{n+1}(X)=\sum_{o\in O}C_n(X \text{ in } v_o/E_o \text{ in } v_o)C_{n+m}(E_o\&V_o)$ if $\{E_o\&V_o\}_{o\in O}$ is logically optimal and sufficiently inclusive for X.

Let $\{E\&V\}$ be B's time-observation partition from t_n to t_{n+1}. Then,

(RSSC) $C_{n+1}(X)=C_n(X \text{ in } v/E \text{ in } v)$ if $\{E\&V\}$ is logically optimal and sufficiently inclusive for X.

These principles provide convenient shortcuts for the cases where the direct application of GSJC is awkward.

When restricted in this way, I believe that SSJC, SSSC, SJC, and SSC become more safe and attractive rules. As such, GSJC subsumes the intuitions behind the attractive instances of these rules.

I. Conclusion

At this point, it will be useful to recapitulate the discussions I have presented thus far in this dissertation. In Chapter II, I developed an updating rule, SJC, which provides an elegant solution for the SB problem. In Chapter III, I generalized that rule for the credal transitions between epistemic moments that are not necessarily contiguous. Unfortunately, the resulting rule, SSJC, yielded different results depending upon whether we apply it to SB's credal transitions step-by-step (from *s* to *m* and then from *m* to *m*+) or all at once (from *s* to *m*+). In this chapter, I presented a rule free from such inconsistency, at least regarding the SB problem. If I had suggested modifying SSJC to GSJC only to avoid the inconsistent results, I would not be able to avoid the charge of *adhocery*; however, I have avoided this charge by providing independent reasons for such modification.

CHAPTER V

SATISFACTION OF DESIDERATA

A. Introduction

In the previous chapters, I discussed a series of updating rules. In each chapter, I suggested a rule for updating one's *de nunc* credences (or the degrees of tensed beliefs). In each case, however, a problem was discovered. Facing each problem, I responded by suggesting an enhanced rule, immune to the newly discovered problem. In addition, I provided an independent reason to favor the enhanced rule, on the basis of Gaifman's Expert Principle.

In chapter IV, I presented GSJC as the final product of this process. It supports some instances of SJC and SSJC, which meet a few special conditions. If we restrict SJC and SSJC with those conditions as provisos, as I believe that we should, we may consider them to be subordinate rules of GSJC. As such, GSJC is the most general rule discussed so far.

This raises a question: Is GSJC the most general rule for *de nunc* updating, full-stop? In other words, is GSJC a rule such that it is *always* rational to update one's *de nunc* credences in accordance with it (hereafter: the Final Rule)?[61]

Recalling my trial and error in the earlier chapters, I find this question to be hard to answer with confidence. For how can we rule out the possibility that GSJC also suffers

[61] Of course, other conditions will have to be satisfied. For example, I will assume that the agent has a perfect memory about her own past opinions. In this chapter, I will assume that the given agent satisfies such basic conditions.

from a now unknown but serious problem and that there exists a better rule for updating, which is immune to that problem and defensible for other reasons? Of course, we cannot.

Nevertheless, GSJC *might be* the Final Rule. Think about the following properties, which any acceptable rule for updating will satisfy. First, the Final Rule will produce only *synchronically coherent* credence functions that *diachronically cohere* with the earlier ones: Suppose that the agent has always updated her *de nunc* credences in accordance with the Final Rule. Then, her resulting present credence function will satisfy the standard axioms of probability, and it will be rationally related to any past credence distribution of the same agent's. Second, the Final Rule will be *observationally exhaustive*: If the agent has updated her *de nunc* credences in accordance with the Final Rule, the present credal judgments resulting from such updates will incorporate the totality of what she has observed until now.

In this chapter, I argue that GSJC satisfies these two requirements. Hence, GSJC is not disqualified as a candidate for the Final Rule; at least, it is not disqualified due to any failure to satisfy those requirements. This may not seem significant, but remember that I already offered a general argument for GSJC. Plus, it generates plausible instances for various types of credal transitions. In my opinion, when combined, these facts form good evidence that GSJC is very close to the Final Rule, if not identical.

B. Background

Before I begin the main discussion, I will clarify several notions and assumptions that I will use and make in this chapter.

First, I clarify the notions of *synchronic* and *diachronic coherences*: To some extent, these words are self-explanatory: An agent's credence function at t_n is *synchronically coherent* iff the elements of that function cohere with one another, and it *diachronically coheres with* the earlier credence functions iff it is rationally related to the agent's credence functions at $t_{n\text{-}1}$, $t_{n\text{-}2}$, and so on (up to t_0). However, the real challenge is to provide substantial criteria for such coherence, within the same credence function and between different credence functions at different times.

If we put aside a few thorny matters (such as the status of Countable Additivity as an axiom), it is relatively easy to find the criterion for synchronic coherence: An agent's credence function C_n at t_n is synchronically coherent iff C_n satisfies Non-negativity, Normality, and Addivity.[62] Regarding *de dicto* credences, a comparable criterion will be SC or JC: An agent's credence function C_n diachronically coheres with her earlier credence functions iff C_n is related by SC/JC to each of $C_{n\text{-}1}$, …, C_0. Concerning *de nunc* credences, a comparable criterion will be, hopefully, GSJC: An agent's credence function C_n diachronically coheres with the earlier credence functions iff C_n is related by GSJC to each of $C_{n\text{-}1}$, …, C_0.

Second, I discuss how to apply the notion of transitivity to a rule for updating. We all know what transitivity is: For any binary relation R and its field S, R is transitive iff for any $x,y,z \in S$, if x is related by R to y and y is related by R to z, then x is related by R to z. However, it is not so obvious that a rule for updating captures a binary relation. Typically, such a rule relates more than two things. A rule for updating describes the relation between *two* credence functions, but its relata include other entities as well. For example, SC/JC seems to describe the relation among (i) the agent, (ii) her observations at the given moment, and (iii) her old and new credence functions. However, we can

[62] For simplicity, I shall use "Additivity" to refer to Finite Additivity in this chapter. However, everything discussed here would apply in the same way if I were to use that word to refer to Countable Additivity.

focus upon a fixed agent and her history of experiences. With those other elements fixed, we can regard SC/JC as (capturing) a binary relation between two credence functions. (Compare: The ancestor-descendant relation appears to have a family as one of the related entities, but we can treat it as a binary relation by considering only a fixed family line.) By using a similar method, we can define transitivity for updating principles: Focus upon a fixed agent *B* (having such and such an epistemic history *H*). Then, an updating rule *R* is transitive iff for any credence functions C_n, C_{n+m}, C_{n+m+l} of *B*'s, if C_{n+m} is related by *R* to C_n, and C_{n+m+l} is related by *R* to C_{n+m}, then C_{n+m+l} is also related by *R* to C_n.

Third, I discuss the notion of "epistemic kernel rules for updating": As I mentioned above, a rule for updating describes a relation connecting (at least) an agent's credence functions at different moments. Usually, neither of those credence functions is assumed to play a special role in that relation. For example, think about "$C_{n+1}(X)=C_n(X/E)$" as an instance of SC. In the credal transition described here, we can say that C_n is the source and C_{n+1} is the result but, most likely, C_n was the result of the given agent's previous credal transition, and C_{n+1} will be the source for her next credal transition. In this sense, C_n and C_{n+1} are not so different in their roles. Let's call this category of rule "transitional rules for updating."

Interestingly, Meacham (2008; forthcoming) suggests a different type of rule for updating, which he calls "epistemic kernel rules for updating." According to him, an epistemic kernel rule for updating relates an agent's ordinary credence function to a special credence function, which he calls "the kernel." When compared with ordinary credence functions, a kernel is supposed to play a special role. For example, here is an epistemic kernel version of SC: $C_n(X)=C_0(X/\mathcal{TE})$ for any proposition *X*, where C_0 is the agent's kernel and $\mathcal{TE}$ is the totality of her observations until t_n. Note that every instance of this rule includes C_0 as the source and never includes it as the result. In this sense, C_0 plays a different role from that of any other credence function of the same agent's.

Exactly what is this so-called kernel? It is difficult to say, because Meacham does not provide a very clear definition for that notion. Here is my best shot: Your kernel is a credence function that you would have if you were stripped of all the data from your past and present observations. For this reason, if you update your credences in accordance with an epistemic kernel rule, it will transform your kernel into a new credence function, incorporating the totality of all your observations whatsoever until now, not just the totality of your observations after some earlier epistemic moment.

While this is an interesting idea, it comes at a cost. First, one may complain that an agent's observations might be so crucial for making any credal judgment that if she were stripped of them, she would not have any remaining credal opinions. Hence, there is no guarantee that every agent has a kernel in the above sense. Second, "if she were stripped of all her observational data" seems to be figurative language. How would we express the above idea more literally? I am not sure, and I suspect that many people will feel the same way.

To avoid these problems, I adopt this approach: I will consider a set of special agents, those who had their *first* credal opinions. Hence, each of them is *guaranteed to have* an initial credence function C_0, and C_0 was *literally* formed without the help of any observational data. By considering only those having these features, we can avoid the two problems mentioned in the previous paragraph.

Of course, this approach has its own cost: If we adopt this approach, our discussion about epistemic kernel rules will be restricted to the credence functions had by these special agents. In my opinion, this is a quite acceptable cost. For first, almost every possible agent must have had an initial credence function at some point in her life, except those rare agents who have lived for an eternal time with no beginning and, for those not in this elite group, their initial credences are likely to have been based upon very few observations. Moreover, second, what we can learn from this potentially narrow range of agents may include important lessons applicable to a broader range of agents, especially about a special topic such as the rational rule for updating *de nunc* credences.

In this section, I have clarified the notions and assumptions that I will use moving forward. Having done this, I am now ready for the main discussions.

C. Strategy

Earlier, I suggested that if an agent has updated her credences in accordance with the Final Rule, first, her resulting credence function will be *synchronically coherent* and it will *diachronically cohere* with her earlier credence functions, and second, as a form of judgment, that credence function will incorporate *the totality of her observations* until the present moment.

Why do I think that the Final Rule will satisfy these requirements? It is clear why a rational rule for updating will produce only synchronically coherent credence functions: For any single credence function of yours, you want its elements to be rationally related. The standard axioms of probability are meant to capture exactly this relation (in the domain of subjective probabilities).

It is also clear why a rational rule for updating will produce only a credence function that diachronically coheres with any earlier credence function: For any two credence functions of yours, you want one of them to be rationally related to the other. For even if your credence function is internally coherent at any fixed time, you will be regarded as an unacceptably whimsical agent unless your credence function at each moment is rationally related to those at the earlier moments. The standard rules for *de dicto* updating, strict and Jeffrey conditionalizations, are meant to capture this relation. In this dissertation, I am trying to find a similar rule for *de nunc* updating.

How about the requirement of observational exhaustiveness? Many mainstream epistemologists will agree that an agent's judgment needs to be based upon what she has observed. The following thesis, especially, is popular in the literature: An agent's belief is justified iff that belief is supported by her evidence. (Connee and Feldman (2004) offer a similar thesis). I also find it to be an intuitive claim.

However, we should be careful. First, in the literature, "evidence" has been used frequently with the connotation that its referent is easily accessible to the given agent (hereafter: accessibility connotation). Perhaps, we can formulate a similar thesis without being committed to this connotation or sacrificing the intuitive appeal of the above thesis. For this formulation, I suggest using "observation" to refer to what plays a similar justificatory role to that of evidence but does not have the accessibility connotation.

Second, even if an observation E supports a hypothesis H when considered in separation, the rest of one's observations may include other information that undercuts or rebuts E's support for H (Pollock & Cruz, 1986). Hence, if we judge whether an agent's belief is justified by considering only fragments of her observations, it may lead to a disastrous result. Therefore, we should make such a judgment by considering the totality of the given agent's observations.

These considerations lead to the following refinement of the above thesis: An agent's belief is justified iff that belief is justified by the totality of her observations. Can we formulate an analogous thesis for probabilism? Yes: An agent's degree of belief is justified iff that degree of belief is supported by the totality of her observations. This thesis is plausible for the same reason as its counterpart in the mainstream epistemology: Intuitively, a rational agent's credal judgment needs to be based upon her observations, and if she makes such a judgment on the basis of only a proper subset of her observations, the rest of her observations may include what would have resulted in a different credal judgment if considered in making that judgment. In addition, the above thesis is compatible with the updating model based on JC, in which the agent is not assumed to have infallible access to her own observations.[63]

For these reasons, I believe that any ideal general rule for *de nunc* updating will satisfy the two mentioned requirements. Of course, even if a certain rule for *de nunc* updating, say, *R*, satisfies those requirements, it is still possible that *R* fails to satisfy some other crucial requirements, of which I am not aware yet. However, *R*'s satisfaction of the discussed requirements will certainly provide some reason to suspect that *R* is the general rule for *de nunc* updating.

In the rest of this chapter, I shall proceed in the following order: In Section D, I will present GSJC and GSR in yet new forms. As formulated in these new forms, they

[63] Another merit is its compatibility with justificatory externalism, but this does not look like a significant merit for the standard probabilism, which already assumes a psychological (read: "inside the skull") account of beliefs. However, see Williamson (2002) for an externalist version of probabilism.

will be called "GSJC$^+$" and "GSR$^+$." In Section E, I will prove that GSR$^+$ captures a transitive binary relation between credence functions, a crucial lemma for the rest of the chapter. In Section F, I will show that GSJC$^+$ satisfies the first requirement: If you have always updated your credences in accordance with GSJC$^+$, your resulting credence function will be synchronically coherent, and it will diachronically cohere with your earlier credence functions. In Section G, I will argue that GSJC$^+$ satisfies the second requirement: Under the same assumption, we can show that your credence functions resulting from such updates will incorporate the totality of your past observations. In Section H, I will point out a difference between GSJC, the earlier formulation of my general updating rule, and GSJC$^+$, the new formulation of that rule presented in this chapter. I will suggest a way to fill this gap.

D. GSJC$^+$ and GSR$^+$

In this section, I present GSJC and GSR again, but this time, in a more technically rigorous way. After that, I discuss their logical relations.

In presenting GSJC and GSR in new forms, I put two restrictions on them: First, I put a restriction on how the domain of credence functions is constructed. In the earlier chapters, I didn't impose any restriction on it except that it consists of tensed propositions. However, it is usual to assume *algebra* made out of propositions, events, or sets as the domain of credence functions. Since we are dealing with tensed propositions, let Γ be the class of all tensed propositions, and let Ω be an algebra made out of *S*, i.e., Ω is a subset of the power set of Γ closed under conjunction and complementation. Additionally, we assume that Ω is also closed under [… at τ] and [… in ν] operations, i.e., if X is a member of Ω, then so are [X at τ] and [X in ν], where "τ" is any term referring to a moment, and "$\mathcal{V}$" is any term referring to an interval. Next, let Δ be the set of an agent *B*'s credence functions. We assume that for any $C\in\Delta$, the domain of *C* is Ω.

Second, I put a restriction on the time intervals and time-specifying tensed propositions, which will appear in my new formulations of GSJC and GSR. Consider partitions Φ and $\Psi\subseteq\Omega$, which meet the following conditions: (i) Φ consists of temporal intervals, and Ψ consists of tensed propositions specifying what time it is. (ii) For any $v\in\Phi$, Ψ includes *V* or the tensed proposition that it is *v* now. Conversely, for any $V\in\Psi$, Φ includes a minimal interval throughout which *V* is true. (iii) For any $X\in\Omega$ and $v\in\Phi$, the truth-value of *X* is invariant within *v*. (iv) Let *X* and *Y* be any genuine propositions that belong to Ω. Suppose that $r=C(X/Y\&\mathcal{R})$, where $C\in\Delta$ and $\mathcal{R}=(W^1$ at $\text{prev}_0)\&(W^2$ at $\text{prev}_1)\&\ldots\&(\mathcal{W}^m$ at $\text{prev}_\text{n})$ for some $<\mathcal{W}^1, \mathcal{W}^2,\ldots, \text{W}^\text{n}>\in\Psi^\text{n}$. Hence, $\mathcal{R}$ is a temporal description or a tensed proposition thoroughly describing what times it has been until now. In this case, even if we replace R with a better temporal description R*, still $\text{C}(X/Y\&\mathcal{R}^*)=r$.[64]

(To see what this means practically, suppose that $r=C_n(X$ in $v^m/\mathcal{D}\&\mathcal{R})$, where $C_n\in\Delta$, $X\in\Omega$, $\mathcal{D}=(E^1$ in $v^1)\&\ldots\&(E^m$ in $v^m)$ for some $<E^1\ldots,E^m>\in\Omega^m$ and $<v^1\ldots,v^m>\in\Phi^m$, and $\mathcal{R}=(W^1$ at $\text{prev}_0)\&\ldots\&(W^m$ at $\text{prev}_{n+1})$ for some $<W^1,\ldots,W^{n+1}>\in\Psi^{n+1}$. By (iii), the truth-value of *X* is invariant within v^m, that of E^1 is invariant within v^1, …, and that of E^m is invariant within v^m. By (iv), $C_n(X$ in $v^m/\mathcal{D}\&\mathcal{R})$ is conditioned upon a well-specified

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[64] Remember the definition of *better de priori information* in the last chapter: Let $\mathcal{R}$ be $(W^1$ at $\text{prev}_0)\&\ldots\&(W^m$ at $\text{prev}_n)$ and $\mathcal{R}^*$ be $(W^{*1}$ at $\text{prev}_0)\&\ldots\&(W^{*m}$ at $\text{prev}_n)$. Let $w^1,\ldots,w^\text{n}$ be intervals associated with $W^1,\ldots,W^\text{n}$; similarly for $w^{*1},\ldots,w^{*\text{n}}$. Then, $\mathcal{R}^*$ is a *better temporal description* than $\mathcal{R}$ iff (i) for every $k\in\{1,\ldots,n\}, w^k\supseteq w^{*k}$ and (ii) for some $k\in\{1,\ldots,n\}$, $w^k\supset w^{*k}$.

temporal description. Clearly, this allows us to do away with the provisos of optimality in the earlier formulations of GSJC and GSR.)

Whenever Δ, Ω, Φ, and Ψ satisfy these conditions, I will say that $<\Delta,\Omega,\Phi,\Psi>$ is a model for *B*'s (*de nunc*) credences. Given a model $<\Delta,\Omega,\Phi,\Psi>$ for *B*'s credences, we are ready to formulate the wanted principles: As before, we let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be an agent *B*'s general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k_o$ at $\text{prev}_{m-k})$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k_o$ at $\text{prev}_{m-k})$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}(W^k_o$ at $\text{prev}_{m+k-1})$. The difference is that this time, we assume that, for each $o\in O$, $<E^1_o,\ldots E^m_o>\in\Omega^m$, $<V^1_o,\ldots,V^m_o>\in\Psi^m$, and $<W^1_o,\ldots,W^n_o>\in\Psi^n$. (Hereafter, I will say that $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is *constructed from* Ω and Ψ whenever it meets these conditions.) Let C_n, $C_{n+m}\in\Delta$. Consider any $X\in\Omega$. Then, I make these claims:

(GSJC$^+$) $\quad C_{n+m}(X)=\sum_{o\in O}C_n(X \text{ in } v^m_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$.

(GSR$^+$) $\quad C_{n+m}(X/\ \mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(X \text{ in } v^m_o/\mathcal{D}_o\&\mathcal{R}_o)$ for each $o\in O$.

Here, $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$, and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the *m* epistemic moments earlier time (i.e., $\mathcal{D}_o=\&_{1\leq k\leq m}(E^k_o$ in $v^k_o)$ and $\mathcal{R}_o=\&_{1\leq k\leq n+1}(W^k_o$ at $\text{prev}_{k-1}))$.

Next, I discuss the logical relation between GSJC$^+$ and GSR$^+$: Let C_n and C_{n+m} be B's credence functions at t_n and t_{n+m}. (Hence, C_n, $C_{n+m} \in \Delta$.) Then, we can prove these two facts:

(1) If C_{n+m} is synchronically coherent and related by GSR$^+$ to C_n, then C_{n+m} is also related by GSJC$^+$ to C_n.

(2) If C_n is synchronically coherent and C_{n+m} is related by GSJC$^+$ to C_n, then C_{n+m} is also related by GSR$^+$ to C_n.

(See APPENDIX B for the proofs.) It is true that these simply mean that GSJC$^+$ is equivalent to GSR$^+$ because a rational agent's credence function always will be synchronically coherent. But it is one of my goals in this chapter to show that if you always update in accordance with GSJC$^+$, all your credence functions will be synchronically coherent. Hence, I do not want to assume the synchronic coherence of B's credence functions from the outset.

Still, this means that those principles will *turn out to be* equivalent, if we independently prove the synchronical coherence of all B's credence functions. Assuming such an independent proof, it will be possible to use GSR$^+$ as a *proxy* for GSJC$^+$: We can show that GSJC$^+$ satisfies a wanted requirement by showing that GSR$^+$ satisfies it.

E. The Transitivity of GSR$^+$

In this section, I argue that GSR$^+$ captures a transitive binary relation between one's credence functions at various moments. For an easier discussion, I will first prove an analogous claim for Rigidity, and then provide a proof for the transitivity of GSR$^+$, by modifying the first proof.

First, I prove the transitivity of (the *de dicto* version of) Rigidity: If Rigidity holds between C_n and C_{n+m} and between C_{n+m} and C_{n+m+l}, then it also holds between C_n and C_{n+m+l}. To prove this claim, we first suppose these conditions: Let $\{E_i\}_{i\in I^*}$ be a partition such that each E_i describes a possible course of B's observations during $[t_{n+1},t_{n+m}]$. Consider $\{E_i\}_{i\in I}$ such that $I\subseteq I^*$, $C_{n+m}(E_i)>0$ and $\sum_{i\in I} C_{n+m}(E_i)=1$. In such a case, we will call $\{E_i\}_{i\in I}$ "(B's) observation partition from t_n to t_{n+m}." Let $\{F_j\}_{j\in J}$ be also B's observation partition from t_{n+m} to t_{n+m+l} in a similar sense. We suppose that

(3) $C_{n+m}(X/E_i)=C_n(X/E_i)$ for any proposition X and any $i\in I$, and

(4) $C_{n+m+l}(X/F_j)=C_{n+m}(X/F_j)$ for any proposition X and any $j\in J$

and show that

(5) $C_{n+m+l}(X/G_k)=C_n(X/G_k)$ for any proposition X and for any $k\in K$,

where $\{G_k\}_{k\in K}$ is B's observation partition from t_n to t_{n+m+l}.

To show this, we need to know how $\{G_k\}_{k\in K}$ is related to $\{E_i\}_{i\in I}$ and $\{F_j\}_{j\in J}$. I claim that each G_k should be $E_i\&F_j$ for some $<i,j>\in I\times J$. For remember that each E_i describes a possible course of observations during $[t_{n+1},t_{n+m}]$, and each F_k describes a possible course of observations during $[t_{n+m+1},t_{n+m+l}]$. Since each G_k represents a possible course of observations during $[t_{n+1},t_{n+m+l}]$, $G_k=E_i\&F_j$ for some $<i,j>\in I\times J$. For convenience, we define $E_k=E_i$ and $F_k=F_j$ when $G_k=E_i\&F_j$. So it suffices to show

(6) $\quad C_{n+m+l}(X/E_k\&F_k)=C_n(X/E_k\&F_k)$ for any proposition X and any $k\in K$.

This is easy to prove: Let X be any genuine proposition. Then, we can derive the following facts:

(7) $\quad C_n(X/E_k\&F_k)=C_n(X\&F_k/E_k)/C_n(F_k/E_k)$,

(8) $\quad C_{n+m}(X\&F_k/E_k)/C_{n+m}(F_k/E_k)=C_{n+m}(X\&E_k/F_k)/C_{n+m}(E_k/F_k)$, and

(9) $\quad C_{n+m+l}(X\&E_k/F_k)/C_{n+m+l}(E_k/F_k)=C_{n+m+l}(X/E_k\&F_k)$.

By (3), $C_n(X\&F_k/E_k)=C_n(X\&F_j/E_i)=C_{n+m}(X\&F_j/E_i)=C_{n+m}(X\&F_k/E_k)$ and $C_n(F_k/E_k)= C_n(F_j/E_i)=C_{n+m}(F_j/E_i)=C_{n+m}(F_k/E_k)$. By (4), $C_{n+m}(X\&E_k/F_k)=C_{n+m}(X\&E_i/F_j)=C_{n+m+l}(X\& E_i/F_j)=C_{n+m+l}(X\&E_k/F_k)$ and $C_{n+m}(E_k/F_k)=C_{n+m}(E_i/F_j)=C_{n+m+l}(E_i/F_j)=C_{n+m+l}(E_k/F_k)$. Done.

Second, I prove GSR^{+}'s transitivity. My proof of this fact will be structurally similar to that of the transitivity of Rigidity, although more complex: Let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be B's general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})$. Also, let $\{\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p\}_{p\in P}$ be B's general time-observation partition from t_{n+m} to t_{n+m+l}, where $\mathcal{F}_p=\&_{1\leq k\leq l}(F^k{}_p \text{ at prev}_{l-k})$, $\mathcal{V}_p=\&_{1\leq k\leq l}(V^k{}_p \text{ at prev}_{l-k})$, and $\mathcal{W}_p=\&_{1\leq k\leq n+m+1}(W^k{}_p$ at prev$_{l+k\text{-}1}$). Both are constructed from Ω and Ψ. Then, we suppose that GSR$^+$ holds between C_n and C_{n+m} and between C_{n+m} and C_{n+m+l}. In other words,

(10) $C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n\,(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)$ for any tensed proposition X and $o\in O$, and

(11) $C_{n+m+l}(X/\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)=C_{n+m}\,(X \text{ in } v^l{}_p/\mathcal{D}_p\&\mathcal{R}_p)$ for any tensed proposition X and $p\in P$,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$, and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the m epistemic moments earlier time, and $\mathcal{D}_p$ is the sequential de-indexicalization of $\mathcal{F}_p$ under $\mathcal{V}_p$, and $\mathcal{R}_p$ is the sequential re-indexicalization of $\mathcal{W}_p$ for the l epistemic moments earlier time. From these suppositions, we prove that GSR$^+$ holds between C_n and C_{n+m+l}. Let $\{\mathcal{G}_q\&\mathcal{V}_q\&\mathcal{W}_q\}_{q\in Q}$ be B's general time-observation partition from t_n to t_{n+m+l}, where $\mathcal{G}_q=\&_{1\leq k\leq m+l}(G^k{}_q$ at prev$_{m+l\text{-}k})$, $\mathcal{V}_q=\&_{1\leq k\leq m+l}(V^k{}_q$ at prev$_{m+l\text{-}k})$, and $\mathcal{W}_q=\&_{1\leq k\leq n+1}(W^k{}_q$ at prev$_{m+l+k\text{-}1})$. We want to show that

(12) $C_{n+m+l}(X/\mathcal{G}_q\&\mathcal{V}_q\&\mathcal{W}_q)=C_n(X \text{ in } v^{m+l}{}_q/\mathcal{D}_q\&\mathcal{R}_q)$ for any tensed proposition X and $q\in Q$,

where $\mathcal{D}_q$ is the sequential de-indexicalization of $\mathcal{G}_q$ under $\mathcal{V}_q$, and $\mathcal{R}_q$ is the sequential re-indexicalization of $\mathcal{W}_q$ for the $m+l$ epistemic moments earlier time.

As before, the key to the proof is in finding the correct relation of $\{\mathcal{G}_q\&\mathcal{V}_q\&\mathcal{W}_q\}_{q\in Q}$ to $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ and $\{\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p\}_{p\in P}$. Here is that relation: For any $q\in Q$, there exists $\langle o,p\rangle\in O\times P$ such that

(13) $\mathcal{G}_q=\mathop{\&}_{1\leq k\leq m}(E^k{}_o \text{ at } \mathrm{prev}_{m+l-k})\&\mathop{\&}_{1\leq k\leq l}(F^k{}_p \text{ at } \mathrm{prev}_{l-k})$;

(14) $\mathcal{V}_q=\mathop{\&}_{1\leq k\leq m}(V^k{}_o \text{ at } \mathrm{prev}_{m+l-k})\&\mathop{\&}_{1\leq k\leq l}(V^k{}_p \text{ at } \mathrm{prev}_{l-k})$;

(15) $\mathcal{W}_q=\mathop{\&}_{1\leq k\leq n+1}(W^k{}_o \text{ at } \mathrm{prev}_{m+l+k-1})$ and $\mathcal{R}_q=\mathop{\&}_{1\leq k\leq n+1}(W^k{}_o \text{ at } \mathrm{prev}_{k-1})=\mathcal{R}_o$;

(16) $\mathcal{W}_p=\mathop{\&}_{1\leq k\leq m}(V^k{}_o \text{ at } \mathrm{prev}_{m+l-k})\&\mathop{\&}_{1\leq k\leq n+1}(W^k{}_o \text{ at } \mathrm{prev}_{m+l+k-1})$ and

$\mathcal{R}_p=\mathop{\&}_{1\leq k\leq m}(V^k{}_o \text{ at } \mathrm{prev}_{m-k})\&\mathop{\&}_{1\leq k\leq n+1}(W^k{}_o \text{ at } \mathrm{prev}_{m+k-1})=\mathcal{V}_o\&\mathcal{W}_o$.

(See APPENDIX C for a proof.)

Having established these facts, I introduce the following definitions: For any $q\in Q$, for *the* $\langle o,p\rangle\in O\times P$ such that (13)-(16) hold between o, p, and q,

(17) $\mathcal{E}_q=_{\mathrm{df}}\mathop{\&}_{1\leq k\leq m}(E^k{}_o \text{ at } \mathrm{prev}_{m+l-k})$ and $\mathcal{F}_q=_{\mathrm{df}}\mathcal{F}_p=\mathop{\&}_{1\leq k\leq l}(F^k{}_p \text{ at } \mathrm{prev}_{l-k})$;

(18) $\mathcal{V}^{\mathcal{E}}{}_q=_{\mathrm{df}}\mathop{\&}_{1\leq k\leq m}(V^k{}_o \text{ at } \mathrm{prev}_{m+l-k})$ and $\mathcal{V}^{\mathcal{F}}{}_q=_{\mathrm{df}}\mathcal{V}_p=\mathop{\&}_{1\leq k\leq l}(V^k{}_p \text{ at } \mathrm{prev}_{l-k})$;

and

(19) $\mathcal{D}^{\mathcal{E}}{}_q=_{\mathrm{df}}\mathcal{D}_o=\mathop{\&}_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)$ and $\mathcal{D}^{\mathcal{F}}{}_q=_{\mathrm{df}}\mathcal{D}_p=\mathop{\&}_{1\leq k\leq l}(F^k{}_p \text{ in } v^k{}_p)$,

It follows from (13)-(19) that, for any $q\in Q$,

(20) $\mathcal{G}_q=\mathcal{E}_q\&\mathcal{F}_q$, $\mathcal{V}_q=\mathcal{V}^{\mathcal{E}}_q\&\mathcal{V}^{\mathcal{F}}_q$, and $\mathcal{D}_q=\mathcal{D}^{\mathcal{E}}_q\&\mathcal{D}^{\mathcal{F}}_q$; and

(21) $\mathcal{W}_p=\mathcal{V}^{\mathcal{E}}_q\&\mathcal{W}_q$.

Consider any $X\in\Omega$. By (19), it suffices to show that

(22) $C_{n+m+l}(X/\mathcal{E}_q\&\mathcal{F}_q\&\mathcal{V}^{\mathcal{E}}_q\&\mathcal{V}^{\mathcal{F}}_q\&\mathcal{W}_q)=C_n(X\text{ in }v^{m+l}_q/\mathcal{D}^{\mathcal{E}}_q\&\mathcal{D}^{\mathcal{F}}_q\&\mathcal{R}_q)$.

To show it, first note that

(23) $\mathcal{E}_o\&\mathcal{V}_o$ is equivalent to $\mathcal{D}_o\&\mathcal{V}_o$, and $\mathcal{D}^{\mathcal{E}}_q\&\mathcal{V}^{\mathcal{E}}_q$ is equivalent to $\mathcal{E}_q\&\mathcal{V}^{\mathcal{E}}_q$.[65]

From the above definitions and facts, it follows that for any $X\in\Omega$,

[65] In showing the first equivalence, we are trying to show the equivalence between $\&_{1\le k\le m}(E^k_o\text{ at prev}_{m+l-k})\&\&_{1\le k\le m}(V^k_o\text{ at prev}_{m+l-k})$ and $\&_{1\le k\le m}(E^k_o\text{ in }v^k_o)\&\&_{1\le k\le m}(V^k_o\text{ at prev}_{m+l-k})$. Clearly, it suffices to show that, for any number $k\in\{1,...,m\}$, $(E^k_o\text{ at prev}_{m+l-k})\&(V^k_o\text{ at prev}_{m+l-k})$ is equivalent to $(E^k_o\text{ in }v^k_o)\&(V^k_o\text{ at prev}_{m+l-k})$. To show this, consider any $k\in\{1,...,m\}$. (=>) Assume $(E^k_o\text{ at prev}_{m+l-k})\&(V^k_o\text{ at prev}_{m+l-k})$ and show $(E^k_o\text{ in }v^k_o)\&(V^k_o\text{ at prev}_{m+l-k})$. By supposition, it was v^k_o at the $m+l-k$ epistemic moments earlier time, and E^k_o was true then. By the construction of Ω and Φ, the truth-value of E^k_o is invariant within v^k_o. Hence, E^k_o is true at any $t\in v^k_o$; by definition, $(E^k_o\text{ in }v^k_o)$ is true. Since the supposition provides the other conjunct, done. (<=) Assume $(E^k_o\text{ in }v^k_o)\&(V^k_o\text{ at prev}_{m+l-k})$ and show $(E^k_o\text{ at prev}_{m+l-k})\&(V^k_o\text{ at prev}_{m+l-k})$. By assumption, E^k_o is true at any moment in v^k_o and it was v^k_o at the $m+l-k$ epistemic moments earlier time. Clearly, it follows that E^k_o was true at the $m+l-k$ epistemic moments earlier time; by definition, $(E^k_o\text{ at prev}_{m+l-k})$ is true. Done. The second equivalence can be shown in a similar way.

(24) $C_n(X \text{ in } v^{m+l}{}_q/\mathcal{D}^{\mathcal{E}}{}_q\&\mathcal{D}^{\mathcal{F}}{}_q\&\mathcal{R}_q)=$

$C_n((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_o)/C_n(\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_o)$,[66]

(25) $C_{n+m}((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_p/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)/C_{n+m}(\mathcal{D}_p/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=$

$C_{n+m}((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_o/\mathcal{D}_p\&\mathcal{R}_p)/C_{n+m}(\mathcal{D}_o/\mathcal{D}_p\&\mathcal{R}_p)$,[67] and

(26) $C_{n+m+l}((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_o/\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)/C_{n+m}(\mathcal{D}_o/\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)=$

$C_{n+m+l}(X/\mathcal{E}_q\&\mathcal{F}_q\&\mathcal{V}^{\mathcal{E}}{}_q\&\mathcal{V}^{\mathcal{F}}{}_q\&\mathcal{W}_q)$.[68]

Thus, it suffices to show that

(27) $C_{n+m}((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_p/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_o)$,

(28) $C_{n+m}(\mathcal{D}_p/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_o)$,

(29) $C_{n+m+l}((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_o/\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)=C_{n+m}((X \text{ in } v^{m+l}{}_q)\&\mathcal{D}_o/\mathcal{D}_p\&$

$\mathcal{R}_p)$, and

(30) $C_{n+m+l}(\mathcal{D}_o/\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)=C_{n+m}(\mathcal{D}_o/\mathcal{D}_p\&\mathcal{R}_p)$.

These facts follow from (10) and (11).[69] Therefore, GSR^+ is transitive.

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[66] For brevity, abbreviate "X in $v^{m+l}{}_q$" into "X^*." Then, $C_n(X^*/\mathcal{D}^{\mathcal{E}}{}_q\&\mathcal{D}^{\mathcal{F}}{}_q\&\mathcal{R}_q)$=(by (15) and (18)) $C_n(X^*/\mathcal{D}_o\&\mathcal{D}_p\&\mathcal{R}_o)=C_n(Z_1\&\mathcal{D}_o\&\mathcal{D}_p\&\mathcal{R}_o)/C_n(\mathcal{D}_o\&\mathcal{D}_p\&\mathcal{R}_o)=C_n(Z_1\&\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_o)/C_n(\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_o)$. Done.

[67] Again, abbreviate "X in $v^{m+l}{}_q$" into "X^*." Then, $C_{n+m}(X^*\&\mathcal{D}_p/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)/C_{n+m}(\mathcal{D}_p/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$=(by (22)) $C_{n+m}(X^*\&\mathcal{D}_p/\mathcal{D}_o\&\mathcal{V}_o\&\mathcal{W}_o)/C_{n+m}(\mathcal{D}_p/\mathcal{D}_o\&\mathcal{V}_o\&\mathcal{W}_o)$=(by (16)) $C_{n+m}(X^*\&\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_p)/C_{n+m}(\mathcal{D}_p/\mathcal{D}_o\&\mathcal{R}_p)=C_{n+m}(X^*\&\mathcal{D}_p\&\mathcal{D}_o\&\mathcal{R}_p)/C_{n+m}(\mathcal{D}_p\&\mathcal{D}_o\&\mathcal{R}_p)=C_{n+m}(X^*\&\mathcal{D}_o/\mathcal{D}_p\&\mathcal{R}_p)/C_{n+m}(\mathcal{D}_o/\mathcal{D}_p\&\mathcal{R}_p)$. Done.

[68] Again, abbreviate "X in $v^{m+l}{}_q$" into "X^*." Then, $C_{n+m+l}(X^*\&\mathcal{D}_o/\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)/C_{n+m}(\mathcal{D}_o/\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)=C_{n+m+l}(X^*\&\mathcal{D}_o\&\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)/C_{n+m}(\mathcal{D}_o\&\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)=C_{n+m+l}(X^*/\mathcal{D}_o\&\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p)$=(by (16)-(18) and (20)) $C_{n+m+l}(X^*/\mathcal{D}^{\mathcal{E}}{}_q\&\mathcal{F}_q\&\mathcal{V}^{\mathcal{E}}{}_q\&\mathcal{V}^{\mathcal{F}}{}_q\&\mathcal{W}_q)$=(by (22)) $C_{n+m+l}(X^*/\mathcal{E}_q\&\mathcal{F}_q\&\mathcal{V}^{\mathcal{E}}{}_q\&\mathcal{V}^{\mathcal{F}}{}_q\&\mathcal{W}_q)$. Done.

[69] Abbreviate "(X in $v^{m+l}{}_q$)&$\mathcal{D}_p$" into "X_1" and "(X in $v^{m+l}{}_q$)&$\mathcal{D}_o$" into "X_2." It follows from (8) and (9) that $C_{n+m}(X_1/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(X_1 \text{ in } v^m{}_q/\mathcal{D}_o\&\mathcal{R}_o)$, $C_{n+m}(\mathcal{D}_p/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(\mathcal{D}_p \text{ in } v^m{}_q/\mathcal{D}_o\&\mathcal{R}_o)$, $C_{n+m+l}(X_2/\mathcal{F}_p\&\mathcal{V}_p$

This is an important result. Using it as a lemma, I will argue in the next two sections that GSR$^+$, when regarded as a rule for *de nunc* updating, satisfies the two aforementioned requirements that must be met by any candidate for the Final Rule.

F. Synchronic and Diachronic Coherence

In this section, I argue that GSJC$^+$ satisfies the first requirement for the Final Rule. In other words, if one's credences have been updated in accordance with GSJC$^+$, the resulting credence function is, first, synchronically coherent in itself, and second, diachronically coherent with any earlier credence function.

For my argument, I assume of an agent *B* that (i) she always updates her credences in accordance with GSJC$^+$. Under this assumption, I will show the synchronic and diachronic coherence of the resulting credence function of *B*'s. To show this, I depend upon other assumptions as well: (ii) Her initial credence function was synchronically coherent. (iii) The credence distribution over her general time-observation partition is always synchronically coherent. Admittedly, neither is trivial, but both are arguably fairly weak assumptions.[70] (Certainly weaker than the results derived from the assumptions.)

First, I show that *B*'s credence function is synchronically coherent at any epistemic moment. Clearly, it suffices to prove that the synchronic coherence of *B*'s

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$\&\boldsymbol{\mathcal{W}}_p$)= $C_{n+m}(X_2$ in $v^l{}_p/\boldsymbol{\mathfrak{D}}_p\&\boldsymbol{\mathfrak{R}}_p)$, and $C_{n+m}(\boldsymbol{\mathfrak{D}}_o/\boldsymbol{\mathfrak{F}}_p\&\boldsymbol{\mathcal{V}}_p\&\ \boldsymbol{\mathcal{W}}_p)=C_{n+m}(\boldsymbol{\mathfrak{D}}_o$ in $v^l{}_p/\boldsymbol{\mathfrak{D}}_p\&\boldsymbol{\mathfrak{R}}_p)$. Since X_1, $\boldsymbol{\mathfrak{D}}_p$, X_2, and $\boldsymbol{\mathfrak{D}}_o$ are all genuine propositions, "in $v^m{}_q$" or "in $v^l{}_q$" is redundant. Done.

[70] I find it especially difficult to explain why one's credence distribution over the general time-observation partition ought to be synchronically coherent. However, note that it is equally hard to explain why one's credence distribution over the observation partition for JC should be synchronically coherent.

credence functions is *preserved* from t_n to t_{n+1}. To prove this, assume that $C_n \in \Delta$ is synchronically coherent, i.e., for any $X,Y \in \Omega$,

(NN$_n$) $C_n(X) \geq 0$,

(NORM$_n$) $C_n(X)=1$ if X is tautological, and

(ADD$_n$) $C_n(X \vee Y)=C_n(X)+C_n(Y)$ if $\sim(X \& Y)$ is tautological.

From this assumption, we prove that $C_{n+1} \in \Delta$ is synchronically coherent, i.e., for any $X,Y \in \Omega$,

(NN$_{n+1}$) $C_{n+1}(X) \geq 0$,

(NORM$_{n+1}$) $C_{n+1}(X)=1$ if X is tautological, and

(ADD$_{n+1}$) $C_{n+1}(X \vee Y)=C_{n+1}(X)+C_{n+1}(Y)$ if $\sim(X \& Y)$ is tautological.

To prove these facts, we will use the following theorems derivable from NN$_n$, NORM$_n$, and ADD$_n$: For any $X,Y,Z \in \Omega$

(CNN$_n$) $C_n(X/Y) \geq 0$ if defined,

(CNORM$_n$) $C_n(X/Y)=1$ if defined and X is tautological, and

(CADD$_n$) $C_n(X \vee Y/Z)=C_n(X/Z)+C_n(Y/Z)$

if defined and $\sim(X \& Y)$ is tautological.

First, I show NN_{n+1}: Let X be any tensed proposition, and let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be B's general time-observation partition from t_n to t_{n+1} (constructed from Ω and Ψ). By (i),

$$(31) \qquad C_{n+1}(X)=\sum_{o\in O}C_n(X \text{ in } v^1{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+1}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o),$$

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$, and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the one epistemic moment earlier time. By (iii) and CNN_n, all terms on the right-hand side are non-negative. Hence, NN_{n+1} is true. Second, I show $NORM_{n+1}$: Consider any X that is tautological. Then, (X in $v^1{}_o$) is also tautological; for, a tautology is logically true at every moment in any temporal interval. By $CNORM_n$, $C_n(X$ in $v^1{}_o/\mathcal{D}_o\&\mathcal{R}_o)=1$ for any $o\in O$. By the definition of the general time-observation partition, $\sum_{o\in O}C_{n+1}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=1$. Hence,

$$(32) \qquad C_{n+1}(X)=\sum_{o\in O}C_n(X \text{ in } v^1{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+1}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=1.$$

Third, I show ADD_{n+1}: Let X and Y be any members of Ω. Suppose that $\sim(X\&Y)$ is tautological. By (i),

$$(33) \qquad C_{n+1}(X)=\sum_{o\in O}C_n(X \text{ in } v^1{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+1}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o),$$

$$(34) \qquad C_{n+1}(Y)=\sum_{o\in O}C_n(Y \text{ in } v^1{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+1}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o), \text{ and}$$

$$(35) \qquad C_{n+1}(X\vee Y)=\sum_{o\in O}C_n((X\vee Y) \text{ in } v^1{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+1}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o).$$

Clearly, [$(X \vee Y)$ in $v^1{}_o$] is equivalent to [$(X$ in $v^1{}_o) \vee (Y$ in $v^1{}_o)$]. Thus,

$$(36)\qquad C_{n+1}(X \vee Y)=\sum_{o\in O} C_n((X \text{ in } v^1{}_o)\vee(Y \text{ in } v^1{}_o)/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)C_{n+1}(\boldsymbol{\mathcal{E}}_o\&\boldsymbol{\mathcal{V}}_o\&\boldsymbol{\mathcal{W}}_o).$$

Because $\sim(X\&Y)$ is a tautology, [$\sim(X\&Y)$ in $v^1{}_o$] is also a tautology. Since [$(X$ in $v^1{}_o)\&(Y$ in $v^1{}_o)$] clearly entails [$(X\&Y)$ in $v^1{}_o$], [$(X$ in $v^1{}_o)\&(Y$ in $v^1{}_o)$] is contradictory; hence, $\sim$[$(X$ in $v^1{}_o)\&(Y$ in $v^1{}_o)$] is another tautology. By CADD$_n$, $C_n((X \text{ in } v^1{}_o)\vee(Y \text{ in } v^1{}_o)/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)=C_n(X \text{ in } v^1{}_o/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)+C_n(Y \text{ in } v^1{}_o/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)$. Therefore,

$$\begin{aligned}(37)\qquad & C_{n+1}(X\vee Y)=\\ & \sum_{o\in O} C_n((X \text{ in } v^1{}_o)\vee(Y \text{ in } v^1{}_o)/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)C_{n+1}(\boldsymbol{\mathcal{E}}_o\&\boldsymbol{\mathcal{V}}_o\&\boldsymbol{\mathcal{W}}_o)=\\ & \sum_{o\in O}[C_n(X \text{ in } v^1{}_o/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)+C_n(Y \text{ in } v^1{}_o/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)]C_{n+1}(\boldsymbol{\mathcal{E}}_o\&\boldsymbol{\mathcal{V}}_o\&\boldsymbol{\mathcal{W}}_o)=\\ & \sum_{o\in O} C_n(X \text{ in } v^1{}_o/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)C_{n+1}(\boldsymbol{\mathcal{E}}_o\&\boldsymbol{\mathcal{V}}_o\&\boldsymbol{\mathcal{W}}_o)+\\ & \sum_{o\in O} C_n(Y \text{ in } v^1{}_o/\boldsymbol{\mathcal{D}}_o\&\boldsymbol{\mathcal{R}}_o)C_{n+1}(\boldsymbol{\mathcal{E}}_o\&\boldsymbol{\mathcal{V}}_o\&\boldsymbol{\mathcal{W}}_o)\\ & =C_{n+1}(X)+C_{n+1}(Y).\end{aligned}$$

In sum, GSJC$^+$ preserves synchronic coherence from C_n to C_{n+1}. Note that I did not depend upon the fact that this is a one-step updating. So GSR$^+$ preserves synchronic coherence from C_n to C_{n+m} for any $m\geq 1$.

Second, I argue that if B updates her credence functions in accordance with GSJC$^+$, then each of her credence functions diachronically coheres with the earlier credence function. For this argument, let us consider any C_n, $C_{n-m}\in\Delta$. I will prove that C_n

is related by $GSJC^+$ to C_{n-m} under (i)-(iii);[71] once this proof is done, it will suffice to argue that C_n diachronically coheres with C_{n-m} if C_n is related by $GSJC^+$ to C_{n-m}.

Here goes the proof: Let $C_n \in \Delta$ be B's credence function at t_n. We want to show that C_n is related by $GSJC^+$ to $C_{n-m} \in \Delta$ for any $m \geq 1$. By (i), C_n is related by $GSJC^+$ to C_{n-1}, C_{n-1} is related by $GSJC^+$ to C_{n-2}, …, and C_1 is related by $GSJC^+$ to C_0. By the earlier result, C_n, C_{n-1}, ..., C_0 are all synchronically coherent. By (1) and (2), $GSJC^+$ and GSR^+ are equivalent for those credence functions. Thus, C_n is related by GSR^+ to C_{n-1}, C_{n-1} is related by GSR^+ to C_{n-2}, …, and C_1 is related by GSR^+ to C_0. By the transitivity of GSR^+, C_n is related by GSR^+ to C_{n-1}, C_{n-2}, …, C_0. By the equivalence, C_n is related by $GSJC^+$ to C_{n-1}, C_{n-2}, …, C_0. Done.

Next, consider this conditional claim: If C_n is related by $GSJC^+$ to C_{n-m}, C_n diachronically coheres with C_{n-m}, i.e., C_n is rationally related to C_{n-m} in the relevant sense. How can I defend this claim? Remember my defense in the last chapter of GSJC as a rational updating rule. According to it, if B sets her credences at t_n by consulting her own credal opinion at t_{n-m}, then GSJC is the right way to do so. This is because, provided that B has made a certain sequence of observations after t_{n-m} but she still considers herself at t_{n-m} as an expert only lacking those observations, there exists a good argument that (a principle entailing) GSJC captures the restriction that B's self at t_{n-m} imposes ? on B's credences at t_n. If this is correct, this restriction will not only justify GSJC as a rule for updating from C_{n-m} to C_n, but it also will justify it as a criterion of diachronic coherence between C_n and C_{n-m}. Since the difference between GSJC and $GSJC^+$ is ignorable here, we have an argument for the wanted claim.

[71] Since (i) is the antecedent of the wanted claim, it will be eventually discharged, but (ii) and (iii) will remain to be substantial assumptions.

Let me combine my discussions in the above two paragraphs: If an agent has always updated her credences in accordance with $GSJC^+$ (and (ii)&(iii) are satisfied), then her present credence function C_n diachronically coheres with any past credence function C_{n-m} of hers.

In sum, I have argued that $GSJC^+$ satisfies the first requirement for the Final Rule. This means that $GSJC^+$ is not ruled out from being the general rule for *de nunc* updating, at least not due to any failure to satisfy this first requirement.

G. Observational Exhaustiveness

In this section, I argue that first, an epistemic kernel version of $GSJC^+$ prescribes an ideal way to incorporate the totality of one's observations into the present credal judgments, and second, the original, transitional version of $GSJC^+$ provides a good way to set one's credences in accordance with its epistemic kernel counterpart.

To begin, I formulate the epistemic kernel version of $GSJC^+$: I assume that an agent B had an initial credence function C_0 at t_0, a moment before she made any observations. Hence, C_0 can play the role of B's kernel, as required for the formulation of an epistemic kernel rule. Next, let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be B's general time-observation partition from t_0 to t_n (constructed from Ω and Ψ), where $\mathcal{E}_o=\&_{1\leq k\leq n}(E^k_o \text{ at prev}_{n-k})$, $\mathcal{V}_o=\&_{1\leq k\leq n}(V^k_o \text{ at prev}_{n-k})$, and $\mathcal{W}_o=(W_o \text{ at prev}_n)$. Given this partition, we can formulate this rule for updating: For any tensed proposition X,

$$(GSJC^{E+})\qquad C_n(X)=\sum_{O\in O}C_0(X \text{ in } v^n_o/\mathcal{D}_o\&\mathcal{R}_o)C_n(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o),$$

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$, and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the n epistemic moments earlier time (i.e., $\mathcal{D}_o = \&_{1\leq k\leq n}(E^k{}_o$ in $v^k{}_o)$ and $\mathcal{R}_o = W_o$ for any $k\in\{1,...,n\}$ and any $o\in O$).

Why does GSJC^{E+} provide a good method for an agent to incorporate the totality of her observations into her credal judgments? First, let's think about a case in which the agent has full knowledge about what observations she has made since t_1 and what times it has been since t_0. In other words, she is fully certain at t_n that she has observed $E^1, E^2, \ldots, E^n$ and that it has been $w, v^1, v^2, \ldots, v^n$ in those orders, for some $<E^1, E^2, \ldots, E^n>\in\Omega^n$ and $<w, v^1, v^2, \ldots, v^n>\in\Phi^{n+1}$. For such a case, GSJC^{E+} will provide this sub-principle: For any tensed proposition X,

(GSSC^{E+}) $\quad C_n(X)=C_0(X \text{ in } v^n/W\&(E^1 \text{ in } v^1)\&(E^2 \text{ in } v^2)\&...\&(E^n \text{ in } v^n))$.

Informally, this simply means that

(GSSC^{E+}) $\quad C_n(X)=$

$C_0(X$ is true during $v^n/$

it is w now&

E^1 is true during v^1&

E^2 is true during v^2&

...

&E^n is true during v^n).

Observe two facts here: First, the above equation seems to capture a very plausible way to incorporate the agent's observations into her credal judgment at t_n. For what could be a more natural way to judge how probable X is at the present moment (which the agent knows to be in v^n) than to judge the probability of "X is true in v^n" conditional on the conjoined initial truth of "it is w now," "E^1 is true during v^1," "E^2 is true during v^2," … and "E^n is true during v^n," when the agent presently knows that this conjunction has been confirmed by her observations so far? Second, and more importantly, those conditions apparently capture *all* of the agent's observations until t_n. Since the agent is assumed here to have observed nothing at t_0, E^1, E^2, …, and E^n exhaust everything that she has observed until the present moment, t_n.

Of course, one may be worried that the agent might not have a full knowledge about what she has observed and/or what times it has been. For such cases, $GSJC^{E+}$ provides a comparably intuitive strategy for epistemic kernel updating: If you do not know what you have observed and/or what times it has been until now, first figure out what credence you would assign to the target tensed proposition if you knew those facts, and next take the weighted average of those credences with the weights being your present credences in various scenarios about your observations and temporal locations until now. Since each of the sequences of observations and times comprising those scenarios exhaust all your observations since t_1, I believe that $GSJC^{E+}$ provides a balanced way to incorporate the *totality* of your observations into the present credal judgments.

If so, it is easy to argue for the next main claim of this section. Remember the earlier result that if an agent *B* has updated her credences in accordance with GSJC$^+$, then *B*'s present credence distribution C_n is related by GSJC$^+$ to C_{n-1}, C_{n-2}, …, C_0. Hence, C_n is related by GSJC$^+$ to C_0 in the mentioned case. But it means that *B*'s credences at t_n are set in accordance with GSJC^{E+}.

This result suggests that if an agent updates her credences in accordance with GSJC$^+$ in the short run and repeats it, she comes to incorporate the totality of her observations (since the initial moment) into her credences in the long run. In my opinion, this is a big merit of GSJC$^+$.

H. Filling the Gap

Remember that I used GSJC in the earlier chapters to solve the problem of Sleeping Beauty. In this section, I first point out that we cannot do the same with GSJC$^+$ because of a new restriction on the general time-observation partition, and I then offer a solution based upon a slightly modified version of GSJC$^+$.

First, think about the following difference between GSJC and GSJC$^+$: While intervals of any size are allowed in the instances of GSJC, only intervals of a very small size are allowed in those of GSJC$^+$. For the members of Φ are such small intervals of time that any better specification of one's temporal location would be meaningless for judging the relevance of the agent's (de-indexicalized) observations to the (de-indexicalized) target tensed proposition. One immediate consequence is that it is difficult to apply GSJC$^+$ to the cases in which time is specified in relatively coarse-grained units. In particular, this means that we cannot solve the Sleeping Beauty problem by using GSJC$^+$. (Certainly, Monday and Tuesday do not belong to Φ.)

To overcome this problem, I suggest yet another variant of GSJC, which I will call "GSJC^0." The main difference between GSJC$^+$ and GSJC^0 is that time is specified by using the members of Φ and Ψ in the former, but it is specified by using the unions or disjunctions of their members in the latter.

To formulate GSJC^0, let $<\Delta,\Omega,\Phi,\Psi>$ be a model for an agent *B*'s credence functions. Then, I will say that $<\Delta,\Omega,\Theta,\Xi>$ is an extension of $<\Delta,\Omega,\Phi,\Psi>$ iff it satisfies the following conditions: (i) If an interval v belongs to Φ, then v also belongs to Θ; if contiguous intervals $v_1,..., v_n$ all belong to Φ, then $\cup_{1\leq j\leq n}v_j$ also belongs to Φ; finally, no other interval belongs to Θ. (ii) Ξ is a superset of Ψ such that if an interval w belongs to Θ, then W or the tensed proposition that it is w now belongs to Ξ, and no other tensed proposition belongs to Ξ. (Note that Φ can include intervals such as Monday and Tuesday; correspondingly, Ξ can contain tensed propositions as those expressed by "it is Monday" and "it is Tuesday.")

Next, let $\{\boldsymbol{\mathcal{E}}_p\&\boldsymbol{\mathcal{V}}_p\&\boldsymbol{\mathcal{W}}_p\}_{p\in P}$ be an agent *B*'s general time-observation partition from t_n to t_{n+m}, where $\boldsymbol{\mathcal{E}}_p=\&_{1\leq k\leq m}(E^k_p \text{ at prev}_{m-k})$, $\boldsymbol{\mathcal{V}}_p=\&_{1\leq k\leq m}(V^k_p \text{ at prev}_{m-k})$, and $\boldsymbol{\mathcal{W}}_p=\&_{1\leq k\leq n+1}(W^k_p \text{ at prev}_{m+k-1})$ for each $p\in P$. But this time, we choose $V^1_p,...,V^m_p$ and $W^1_p,...,W^{n+1}_p$ not from Ψ but from Ξ. (In this case, I will say that $\{\boldsymbol{\mathcal{E}}_p\&\boldsymbol{\mathcal{V}}_p\&\boldsymbol{\mathcal{W}}_p\}_{p\in P}$ is constructed from Ω and Ξ.) The rest of the formulation is similar: For any tensed proposition X,

$$(\text{GSJC}^0)\ C_{n+m}(X)=\sum_{p\in P} C_n(X \text{ in } v^m_p/\mathcal{D}_p\&\mathcal{R}_p)C_{n+m}(\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p)$$

$$\text{if } \{\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p\}_{p\in P} \text{ is optimal for } X,$$

where $\mathcal{D}_p=\&_{1\leq k\leq m}(E^k_p \text{ in } v^k_p)$ and $\mathcal{R}_p=\&_{1\leq k\leq n+1}(W^k_p \text{ at prev}_{k-1})$. Note that we need the explicit proviso of optimality because it is not guaranteed to be satisfied here.

Does GSJC^0 meet the two requirements for the Final Rule? To answer this question, we need to identify the logical relation between GSJC$^+$ and GSJC^0. Let C_n and C_{n+m} be B's credence functions at t_n and t_{n+m}. Then, these claims are true:

(38) If C_{n+m} is related by GSJC^0 to C_n, then C_{n+m} is related by GSJC$^+$ to C_n, and

(39) If C_n and C_{n+m} are synchronically coherent and C_{n+m} is related by GSJC^0 to C_n, then C_{n+m} is also related by GSJC$^+$ to C_n.

(See APPENDIX D for the proofs.)

Once these facts are established, it is easy to argue for GSJC^0's satisfaction of the two requirements for the Final Rule. First, suppose that the given agent, B, has always updated her credences in accordance with GSJC^0 until t_n. Thus, C_n is related by GSJC^0 to C_{n-1}, C_{n-1} is related by GSJC^0 to C_{n-2}, …, C_1 is related by GSJC^0 to C_0. By (38), C_n is related by GSJC$^+$ to C_{n-1}, C_{n-1} is related by GSJC$^+$ to C_{n-2}, …, C_1 is related by GSJC$^+$ to C_0. Since GSJC$^+$ generates only synchronically coherent credence functions in this case, so does GSJC^0. Moreover, it was proven that, in this case, C_n is related by GSJC$^+$ to each of C_{n-1}, C_{n-2}, …, C_0. By (39) and the already proved synchronic coherence of C_n, …, C_1, C_n is related by GSJC^0 to each of C_{n-1}, C_{n-2}, …, C_0. Therefore, GSJC^0 generates a synchronically coherent credence function that diachronically coheres with the earlier credence functions (if GSJC^0 itself is used as the criterion of diachronic coherence). Second, because C_n is related by GSJC^0 to C_0, C_n incorporates the totality of *B*'s observations (if *B* observed nothing at t_0).

Therefore, there exists a principle for updating that satisfies the two requirements for the Final Rule. Furthermore, we can use that rule, GSJC^0, in cases in which time is only coarsely specified, as in the Sleeping Beauty problem.

I. Conclusion

In this chapter, I have argued that if there exists the Final Rule, or a rule that can always be used by a rational agent to update her *de nunc* credences, it will satisfy the two requirements discussed so far. Since GSJC$^+$ (or, equivalently, GSJC^0) was shown to satisfy those requirements, we have a promising candidate for the Final Rule.

CHAPTER VI

CONCLUSION

A. Summary

So far, I have presented and defended GSJC, a new rule for *de nunc* updating. It has various merits, the following being the most notable:

First, GSJC provides a convincing solution for the Sleeping Beauty problem: On the one hand, waking up on Monday seems to be neutral between the coin's landing heads and landing tails. Hence, $C_m(H/W\& MON)=1/2$. On the other hand, waking up on Tuesday entails the coin's landing tails. Thus, $C_m(H/W\&TUE)=0$. If so, SB has to assign the weighted average of ½ and 0 to the coin's landing heads. These facts suggest that $C_m(H)\in(0,1/2)$. I find this line of reasoning to be convincing, and GSJC supports it (under several plausible assumptions).

Second, GSJC applies to a wide range of cases, if not all: In the earlier chapters, I first argued that SJC is the correct rule for *de nunc* updating, at least in some situations, and then generalized the rule and argument together to show that GSJC is the generally correct rule for *de nunc* updating. So GSJC applies to an agent's credal transition from t_n to t_{n+m} for any $m\geq1$. Even if the agent is ignorant of what time it is now or what times it had been until the time from which she is updating, GSJC applies to that credal transition without a hitch.

Third, GSJC is coherent in many aspects: If you feed a synchronically coherent prior credence function to that rule, it spits out another synchronically coherent credence function. Moreover, if you continue to update your credences in accordance with GSJC in the short runs, then you come to have revised your credences in accordance with GSJC in the long run. Plus, it allows you to incorporate all of your accumulated observations into your present credal judgments.

In sum, GSJC provides an intuitive solution for the SB problem, it is applicable to a wide range of cases, and it is coherent in many important aspects. These facts are good reasons to accept GSJC as the general rule for *de nunc* updating.

Although these findings are nice achievements, three important issues are still waiting for our discussion. The first concerns GSJC's generality beyond *de nunc* credences: As it is now, that rule is silent about how to update your *de se* credences in general. The second concerns GSJC's completeness as a rule for updating: If you update in accordance with that rule, you need a predetermined credence distribution over your general time-observation partition. Unfortunately, GSJC is silent about how to acquire such a distribution. The third issue concerns GSJC's complexity: Undoubtedly, it is a highly complex rule. If there exists a simpler rule for *de nunc*/*de se* updating with the same merits, isn't it reasonable to prefer that simpler rule?

In this chapter, I will discuss these issues briefly, but I will refrain from full-fledged discussions. My primary goals in writing this dissertation were, first, to present a promising rule for *de nunc* updating and, second, to develop an argument to defend its adequacy as the general rule for such an updating process. These goals are ambitious enough for the first discussion of any updating rule, and so the more advanced discussions will have to be saved for later papers or books.

In the rest of this chapter, I will proceed in this order: In Section B, I will discuss how to generalize GSJC for *de se* credences. In Section C, I will discuss in more detail how to determine the credence distribution over the general time-observation partition. In Section D, I will discuss the possibility of a simpler rule for *de nunc* or *de se* updating with the same merits as GSJC's, offering some reasons to be skeptical of that possibility.

B. Remaining Issue 1: Generalization for *De Se* Updating

At this point, it is natural to ask this question: "What is the correct rule for *de se* credences in general?" In this section, I suggest a slightly modified version of GSJC as an answer to this question.

Let's begin. As before, let $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ be the agent B's time-observation partition from t_n to t_{n+m}. This time, however, we allow each $\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o$ to include any centered-propositional letters, not just tensed-propositional ones. Then, for any centered proposition X,

(GSJC$_{de\ se}$) $\quad C_{n+m}(X)=\sum_{o\in O} C_n(X \text{ in } v^m{}_o/\mathcal{D}_o \& \mathcal{R}_o)C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)$ if

$\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ is optimal for X, and B is sure at t_{n+m} that C_n was her own credence function m epistemic moments ago,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$, and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the m epistemic moments earlier time. To avoid confusion, I will call the original, tensed version of GSJC "GSJC$_{de\ nunc}$."

What are the differences between $\text{GSJC}_{de\ nunc}$ and $\text{GSJC}_{de\ se}$? The first one is that $\text{GSJC}_{de\ se}$ has centered-propositional letters in the places of tensed-propositional ones in $\text{GSJC}_{de\ nunc}$, and the second one is that $\text{GSJC}_{de\ se}$ has an additional proviso that the agent is presently sure that C_{n+m} is her own credence function at the time from which she is updating.

Set aside the second modification for a moment. Then, I have suggested simply replacing tensed-propositional variables with centered-propositional ones. This suggestion is attractive because scientists and philosophers tend to be conservative: They always want to preserve as many elements of an established theory when they try to generalize it for a broader range of cases. Nevertheless, we need to be careful because this tendency sometimes misfires. For instance, when David Lewis suggested the *de se* version of SC (hereafter: $\text{SC}_{de\ se}$) as the correct rule for *de se* updating, he was obviously being driven by the same kind of conservative tendency. Unfortunately, we know now that $\text{SC}_{de\ se}$ is untenable.

However, I have some hunch that the expansion from $\text{GSJC}_{de\ nunc}$ to $\text{GSJC}_{de\ se}$ will not misfire in a similar way. To understand why, note that we can conceive of three types of beliefs: Those about "what this world is," those about "what time it is now," and those about "who I am." Without a defense, I assume that all beliefs are reducible to these three types of beliefs or the combinations thereof.

The traditional theories of *de dicto* credences took only the first type of beliefs into consideration. That made things easy. For think about this fact: You do not travel from this world to that world, so the truth-values of the first type of beliefs do not change through time. As a result, if you learn *de dicto* evidence E, then you can set your credence in a proposition X by feeding E directly into your previous conditional credence function.[72]

In contrast, my theory of *de nunc* credence covers the second type of beliefs in addition to the first. Unfortunately, this addition called for a drastic modification of the traditional updating rules, which resulted in a much more complex rule, $\text{GSJC}_{de\ nunc}$. The reason was simple: As time flows, you travel from this time to that time. So the future becomes the present, the present becomes the past, and the past becomes the farther past. This fact demands complex techniques for *shifting* or translating your observations or time-specifying propositions into the tensed propositions of the matching truth-values, such as de-indexicalization and re-indexicalization. For example, if you learn "previously, it was 9 AM," you will set your present credence by using your previous credence conditioned upon "it is 9 AM now."

Now, we are talking about how to theorize the third type of beliefs, involving "who I am," in addition to the first and second types. Luckily, it is unlikely to be difficult this time. Why? No one changes from this person to that person! So my hunch says that I won't need a complex technique for shifting in order to update the degrees of my beliefs about "who I am." For instance, if I newly learned (S) "I am the son of Sungki Kim," I will set my present credence by using my previous credence conditioned just upon S. (Analogous to the degrees of *your* beliefs about who you are, are your friend's beliefs, your sister-in-law's, etc.) If this hunch is correct, then it will be okay simply to replace

[72] So $C_{n+1}(X)=C_n(X/E)$. More generally, if your present experience directly affects your credences in E_is, then your present credence in X is acquired by feeding E_i into your previous conditional credence function and taking the weighted average. So $C_{n+1}(X)=\sum_{i\in I}C_n(X/E_i)C_{n+1}(E_i)$.

tensed proposition letters with centered proposition ones within GSJC to get the general rule for *de se* updating.

Still, we need to be careful. Think about this version of the SB problem: **SB problem 4**. On Sunday, SB knows she will go through the following experiment: In the next moment, a group of experimenters will put her to sleep. Then, they will toss a fair coin. Case 1: The coin lands heads. In this case, she wakes up on Monday knowing that it is Monday. Case 2: The coin lands tails. In this case, the experimenters duplicate SB while she is sleeping. Then, they awaken her on Monday, and she knows that it is Monday. What is her credence on Monday in the coin's having landed heads?

In this version of the SB problem, when SB wakes up on Monday, she does not know whether she is SB or the duplicate (hereafter: DUP). Let W be the centered proposition expressed by "I am waking up today with the memory of SB's until Sunday as the last memory." Given the analogy between the two versions of the problem, it is likely that

(40) $C_m(H)=$

$C_s(H/W \text{ is true of SB on Monday})C_m(W\&\text{I am SB})+$

$C_s(H/W \text{ is true of DUP on Monday})C_m(W\&\text{I am DUP})\in(0,1/2).$

In this equation, SB's evidence on Monday, W, is de-indexicalized to "W is true of SB on Monday" or "W is true of DUP on Monday," depending upon who she is. Clearly, this fact contradicts my prior hunch that no shifting technique, such as de-indexicalization, is necessary involving the beliefs about "who I am" because no one changes from this person to that person. What is happening here?

SB problem 4 shows that, although no one can change who she is, a rational agent can sometimes be unsure about whether she is updating from her own previous credence function or somebody else's. From SB's point of view on Monday, she is updating from C_s; so, if she is SB, she is updating from her own previous credence function, but if she is DUP, she is updating from somebody else's previous credence function, namely, SB's.

Here is the general lesson: Usually, a shifting technique such as de-indexicalization is unnecessary for *de se* updating because the agent will know the fact that she is updating from *her own* prior credence function. However, there are rare cases in which a rational agent is unsure of this fact. In such a case, some form of shifting technique will be necessary involving the matter of "who I am." The new proviso, "B is sure at t_{n+m} that C_n was her own credence function…" prevents $\text{GSJC}_{de\ se}$ from erroneously applying to such cases.

In summary, it is acceptable in most of the targeted cases to generalize $\text{GSJC}_{de\ nunc}$ for *de se* credences simply by replacing tensed-propositional letters with centered-propositional ones, but a more complex method for updating will be necessary in some rare cases. The proviso of $\text{GSJC}_{de\ se}$ is a safety device preventing its misapplication to such cases.

C. <u>Remaining Issue 2: Credence Distribution over the Partition</u>

At this point, a brilliant reader already will be asking this question: "How does an agent determine her credence distribution over the general time-observation partition?" In this section, I provide several possible answers.

To begin, I explain what the problem is. For comparison, think about the *de dicto* version of JC (hereafter: $\text{JC}_{de\ dicto}$). According to $\text{JC}_{de\ dicto}$, an agent's present credence in proposition X ought to be the weighted average of her previous credences in X given E_i, where the weights, the agent's present credences in E_is, are somehow "directly affected by" her present observations (Field 1978, p.361; Garber 1980, p.142).

This picture of "probability kinematics" indicates the existence of some relation between the fineness of the observation partition and the agent's perceptual power. For example, suppose that you are watching a piece of cloth under a dim light and it appears to be red or green to you, but you are unsure which color it is. So your observation partition is $\{R, G\}$, where R is "this piece of cloth is red" and G is "this piece of cloth is green." Now you can imagine that if you had better eyesight, then you could distinguish subtler colors, such as pinkish red, yellowish red, yellowish green, and bluish green. In that case, your observation partition would be $\{PR, YR, YG, BG\}$ (where PR is "this piece of cloth is pinkish red," etc.). Observe that this hypothetical observation partition is more fine-grained than your actual observation partition. In general, the stronger your perceptual power is, the finer-grained your observation partition is.

In this sense, your perceptual power sets the limit of the fineness of your observation partition. This fact suggests that, if there exists a more fine-grained partition than your current observation partition, then your credences in the members of that partition won't be directly determined by your observations alone. For that partition will be *too* fine-grained for its members' credences to be determined only by your observations.

If your perceptual power sets the limit of your *de dicto* observation partition's fineness in this sense, it is reasonable to think that the same is true of your *de nunc* observation partition. Unfortunately, this means that at any moment, your credences in your time-observation propositions cannot be directly determined by your observations. For simplicity, let's focus on SJC. According to that rule, $C_{n+1}(X)=C_n(X \text{ in } v_j/E_i \text{ in } v_j)$ $C_{n+1}(E_i\&V_j)$, where C_n and C_{n+1} are your previous and present credence functions. To calculate this value, you need to have a credence distribution over $\{E_i\&V_j\}_{<i,j>\in K}$ at hand. However, your perceptual power at t_{n+1} might be incapable of producing such a credence distribution, if $\{E_i\&V_j\}_{<i,j>\in K}$ is more fine-grained than $\{E_i\}_{i\in I}$.

So we have a problem: Although SJC requires that you have a credence distribution over your time-observation partition at hand, the partition might be too fine-grained for your perceptual experience alone to set the credence distribution over it. This point, of course, generalizes to GSJC; for your general time-observation partition normally will be more fine-grained than your observation partition.

How can we solve this problem? I do not have a fully developed solution yet, but I have been considering three possible solutions. Let me outline them one by one. (Again for simplicity, I will focus upon SJC only, but most of my points below will apply to GSJC as well.)

First, your credence distribution over the time-observation partition may be determined by appealing to a principle of indifference. I already discussed this approach in Chapter II, in which I criticized a principle of indifference endorsed by Adam Elga. In a nutshell, his principle of indifference does not always support a particular credence distribution over your time-observation partition (it does not if a member of the partition spans over an uncountable number of worlds), nor is it guaranteed to be consistent (it leads to a contradiction if the partition includes a possible world with an infinite but countable number of subjectively indistinguishable centered worlds).

Nevertheless, it is still possible that (i) there exists a new version of the principle of indifference, (ii) it provides a general recipe for setting your credence distribution over your time-observation partition, and (iii) it is free from the problem discussed in the last paragraph. If such a principle is found, it will produce a *unique* credence distribution over your time-observation partition. Once such a credence distribution is provided, SJC can do the rest of the job to calculate your credence function over the whole domain.

Sadly, various paradoxes tainted the reputation of the principle of indifference. Here is the common structure of those paradoxes: The principle of indifference applies to a partition, giving the same credence to its members, but in some situations, the same possibilities are divisible into different partitions, leading to conflicting credence assignments. Because of this type of problem, many philosophers have rejected the principle of indifference. Recently, some philosophers have tried to revive it by formulating a new principle of indifference invulnerable to the mentioned paradoxes. (For instance, see Elga (2004), Mikkelson(2004), and White (ms.).) I personally am skeptical of these attempts, but, to be fair, I say that it is too early to make the final judgment. Of course, whether a principle of indifference can solve our problem depends upon the success/failure of this general project.

Second, if you cannot choose *the* right credence function over your time-observation partition, perhaps it will be best not to choose *only one* of them. Instead, you might rather have *all* possible credence distributions compatible with your observations and previous credence function. Many philosophers suggest that, after all, it is unrealistic to represent a human opinion with a single credence function because it is humanly impossible to have such a precise opinion about anything. Instead, they claim that it is better to represent a human opinion at a certain moment with a *set* of credence functions (hereafter: representor) (Sturgeon, 2008; van Fraassen, 1990; Walley, 1991). Accordingly, each proposition in the domain is assigned a *set* of real numbers (hereafter: vague credence). Some advocates of vague credence think that an agent's vague credence in a proposition ought to include *all* compatible values of her credence in that proposition with her data. I suspect that this idea provides a potential solution to our problem.

To see how this idea works, think about SB's credal transition from s to m. Remember that s is SB's last conscious moment on Sunday, and m is the moment of her waking up on Monday. Let CFS_s be her representor at s. For any $C_s \in CFS_s$, $C_s(H)$=1/2 because she knows on Sunday that the coin is fair. Let CFS_m be her representor at m. At m, her time-observation partition is $\{W\&MON, W\&TUE\}$. Since her observations cannot uniquely determine the credences in *W&MON* and *W&TUE*, her vague credence at m in *W&MON* ranges over (0,1).[73] By SJC, $C_m(H)=C_s(H$ on Monday/W on Monday)

[73] Why not [0,1], (0,1] or [0,1)? I tend to think it is crazy that (*) CFS_m includes a function which completely rules out *W&MON* or *W&TUE*, since SB lacks any evidence logically contradicting either of them. However, I admit that I do not have a ready answer to this question: "If (*) is crazy, is it not also insane that CFS_m includes a function that assigns 0.99999… to *W&MON* or *W&TUE*, given that she lacks any evidence supporting either of them to a comparable degree?"

$C_m(W\&MON)$ + C_s(H on Tuesday/ W on Tuesday)$C_m(W\&TUE)$=1/2$C_m(W\&MON)$ for any $C_s \in CFS_s$ and $C_m \in CFS_m$. So her credence value set at m for H ranges over (0,1/2).

In my opinion, this solution is more attractive than the previous one (based on the principle of indifference) for several reasons. First, as the name suggests, the theory of vague credence is nothing more than one application of David Lewis's general theory of vagueness. If anybody is attracted to his theory of vagueness, she also will be attracted to the notion of vague credence. Second, the idea of vague credence is motivated by independent considerations. In reality, nobody can tell what *single real number* she assigns to a proposition as the credence. Hence, if we adopt the notion of vague credence, it will help to build a more realistic model of human credal opinions.

Third, if you are a die-hard subjectivist, you may wonder why there must be a uniquely rational credence distribution, or even a set of credence distributions, over your time-observation partition. To understand this point, think about these facts: In the subjectivist tradition, a rational agent's credal opinion is not assumed to supervene upon her accumulated observational data; to establish supervenience, the agent's initial credence function needs to be included in the supervenience base. In other words, even if perfectly rational agents A and B observe exactly the same data throughout their entire lives, it is possible that A and B will have different credence functions at any time as long as their initial credence functions were different.

How is it possible that A and B's initial credence functions were different? Of course, their initial credal judgments were made before any of their observations. So there could not have been any *a posteriori* constraints on their initial credence functions. Certainly, there must be some *a priori* constraints, but they will not be sufficient to make them have the same initial credence function. Therefore, this policy looks inevitable: Allow *A* to have *any initial credence function that she would like* as long as it is synchronically coherent and it satisfies other rational constraints. *Mutatis mutandis* for *B*.

Perhaps we can say the same thing about an agent *B*'s credence distribution over her time-observation partition. As already emphasized, it is impossible to fully determine this credence distribution if her time-observation partition is more fine-grained than her observation partition. In such a case, perhaps this policy will be unavoidable: Allow *B* to have *any credence distribution over her time-observation partition that she would like* as long as it is synchronically coherent, it satisfies other *a priori* constraints, and it is compatible with her credence distribution over her observation partition. Provided a similar policy for the initial credence function, this *laissez-faire* policy is not so implausible any longer.

So far, I have pointed out a problem regarding how to set your credence distribution over your time-observation partition, necessary for using SJC, and I have outlined three possible solutions to the problem. I do not pretend that these outlined solutions are exhaustive; indeed, I do not find them to be fully satisfactory, and I welcome any new solutions to this problem. If, however, we fail to find a better solution, we can at least return to those outlined here as our fall-back positions. Of course, all these problem and solutions are transferrable to GSJC and general time-observation partition.

D. Remaining Issue 3: The Possibility of a Rival Rule

Admittedly, GSJC is a complex rule. We all hate complex rules; they are hard to understand and difficult to apply to real cases. We would of course prefer a simpler rule for *de nunc* updating, all else being equal. However, in this section I argue that a rival rule satisfying the two conditions—"simpler" and "all else being equal"—will be hard to find, if it exists at all.

The elements making GSJC a complex rule were introduced for some reasons, after all. A GSJC-er will de-indexicalize her observations, which allows her to avoid *the problem of outdated conditional credence*. (Review Chapters II and III.) A GSJC-er will use the sequential de-indexicalization technique to *deal with a sequence of observations*, which allows her to update from a previous, incorrect credence function. (Review Chapter III.) Finally, a GSJC-er re-indexicalizes any *de priori* information she has, which helps to overcome *the problem of impoverished temporal knowledge*. (Review Chapters III and IV.) Clearly, these merits come with the cost of additional complexity. Still, the merits exceed the cost.

Of course, if a simpler rule for *de nunc* updating enjoys all these merits, we will favor such a rival rule over GSJC. I am, however, skeptical of this possibility. After all, such a rival rule also will have to confront the problems mentioned in the last paragraph. As I've shown, these problems can be solved if simplicity is abandoned, but the resulting modification will not be much superior to GSJC in terms of simplicity. In sum, I am concerned that a rival rule either will become equally complex after the necessary modifications are made or will be unable to deal with some of the aforementioned problems.

To illustrate this dilemma, I will discuss an alternative rule for *de se* updating as a case study. Wolfgang Schwarz (ms.) suggests the following rule for *de se* updating: For any centered proposition X, define $>X$ to be that X will be true at the next epistemic moment. For instance, if S is the centered proposition expressed by "I am watching the sun rise," $>S$ will be the centered proposition that S will be true at the next epistemic moment. Using this notation, Schwarz formulates a very simple rule for *de se* updating: Let X be any centered proposition, and let E be the totality of an agent B's observations at t_{n+1}. Then,

(Shifted Conditioning) $\quad C_{n+1}(X)=C_n(>X/>E)$,

where C_n and C_{n+1} are B's credence functions at t_n and t_{n+1}. In words, an agent's present credence in a centered proposition X is equal to her previous credence in X's truth at the next epistemic moment, given E's truth at the next epistemic moment, where E is the totality of her present observations.

Clearly, Shifted Conditioning is a simpler rule than GSJC. However, can a Shifted Conditionalizer deal with the aforementioned problems, which a GSJC-er can handle easily? Not all of them.

First, let's focus on the outdated conditional credence problem. Remember, this problem: If you use Strict Conditionalization to set your present credence in the *present* truth of X, then you come to set it by checking your previous conditional credence in the *previous* truth of X.[74] So, although you are trying to make a credal judgment of whether X is *presently* true, you are doing so by using your previous credal judgment about whether X was *previously* true, where the previous and present moments are different. If X is irreducibly centered, this might be a problem because X's truth-value might have changed.

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[74] For brevity, I omit the probabilistic antecedent of the previous conditional credence used for conditioning, which is your present total evidence E.

At first sight, Shifted Conditioning appears to be free from this problem. For, if you update in accordance with Shifted Conditioning, you come to set your present credence in the *present* truth of *X* by checking your previous conditional credence in the truth at the then *next* moment of *X*.[75] In the last sentence, "present" and "next" are co-referential. As a result, you come to make a credal judgment about whether *X* is *presently* true by using your previous credal judgment about whether *X* would be true at the then *next* moment, just as it should be.

A problem occurs when the agent does not know that the present moment is next to the moment from which she is updating. For example, when SB wakes up on Monday, what will her credence that it is Monday be? According to Shifted Conditioning,

$$C_m(MON)=C_s(>MON/>W)=1.$$

In other words, her credence on Monday in its being Monday is equal to her credence on Sunday night that the next epistemic moment would be on Monday, given that she would wake up (with the memory of Sunday as the last memory) at the next moment. Since she was sure on Sunday that it would be Monday in the next moment, the above instance of Shifted Conditioning implies that waking up on Monday, *she certainly knows that it is Monday*! This is crazy, because she is not in a position to know that it is Monday.

(One obvious escape route is to assume that actually there are two epistemic moments next to the one on Sunday night—one on Monday and the other on Tuesday.

[75] As before, I omit the probabilistic antecedent of the previous conditional credence used for Shifted Conditioning, which is $>E$.

For some curious reason, Schwarz rejects this potential solution.[76] In any case, Shifted Conditioning will have to be greatly modified in order to incorporate such a non-linear structure of nextness among epistemic moments. And that modification will be done only at the cost of increased complexity.)

What is the origin of this problem? On Sunday night, SB knew that the next epistemic moment would be on Monday, whether the coin landed heads or tails. However, when she actually wakes up on Monday, she cannot rule out that it is Tuesday. From her point of view on Monday, if it is Tuesday, the present time is *not* next to the epistemic moment on Sunday night. So she does not know whether or not the present moment is next to the time from which she is updating. Still, Shifted Conditioning forces her to update her credence in *MON as if she knows that the present moment is next to the time from which she is updating*.

In a nutshell, updating in accordance with Shifted Conditioning can be a mistake if the agent is not sure about how her present time is related to the time from which she is updating. So Shifted Conditioning fails to solve the problem of outdated conditioning credence adequately.

Second, Shifted Conditioning is not as versatile as GSJC because it is meant only for the credal transition from the agent's credence function at the previous epistemic moment. Schwarz seems to be aware of this problem and provides a clue as to how to remove this limitation: For any centered proposition X, he defines $>_n X$ to be the centered proposition that X will be true at the n epistemic moments later time. For example, suppose that going to bed, you fully expect to be awakened briefly during the night and to

[76] Schwarz's paper is unclear about this point. In fact, he clarified this point in a private email to me.

be awakened by an alarm in the morning. Let W_A be the centered proposition expressed by "I am being awakened by the alarm." Then, $>_2 W_A$ will be the centered proposition that W_A will be true at the two epistemic moments later time, which you presently know will be the moment of waking up in the morning tomorrow. Using this expanded notation, I formulate what *I* think to be a natural expansion of Shifted Conditioning: Let X be any centered proposition, and let $E^1,\ldots,E^m$ be such that for any $k\in\{1,\ldots,m\}$, E^k is the totality of her observations at t_{n+k}. Then,

(Sequential Shifted Conditioning) $C_{n+m}(X)=C_n(>_m X/\&_{1\leq k\leq m} >_k E^k)$,

where C_n and C_{n+m} are B's credence functions at t_n and t_{n+m}. According to this rule, if an agent has observed E^1, $E^2,\ldots,$ E^m, an agent's present credence in X is equal to her conditional credence at the m epistemic moments earlier time in [X's truth at the m epistemic moments earlier time], given [the conjunction of the truth at the k epistemic moments later time of E^k for all $k\in\{1,\ldots,m\}$]. I suppose this is the best way for Schwarz to go. However, he does not explicitly endorse this extension of Shifted Conditioning, and, even if he did, the cost would be increased complexity. (Also, remember that this modification does not solve the outdated conditional credence problem adequately.)

Thus, as it is now, Shifted Conditioning cannot handle some of the problematic cases with which GSJC has no problem. If you abandon its current simplicity, the thus-modified rule may do better, but Schwarz will have to pay the cost of increased complexity. Of course, I cannot completely rule out the possibility that somebody may find a simpler rule for *de nunc*/*de se* updating, which somehow does not suffer from such problems. However, until I actually find such an alternative rule, I will remain skeptical of that possibility.

E. Conclusion

In this dissertation, I have presented a series of rules for *de nunc* updating. I discussed each of them critically, and, whenever I found a problem, I replaced an earlier rule with a more general and plausible rule for updating. GSJC was the final product of this process. I defended it by appealing to a variant of Gaifman's expert principle, and I showed that it has several highly desirable properties.

In addition, in this chapter I suggested that GSJC might be further generalized to a rule for *de se* updating and complemented by some strategy for setting the credence distribution over an agent's general time-observation partition. Also, I explained why I am skeptical about the possibility of any simpler rule for *de nunc*/*de se* updating enjoying all the merits of GSJC. Given all these results and educated hunches, I believe that GSJC is close to being the general rule for *de se* updating.

If this belief is correct, what will its general ramifications be? Traditionally, the following elements have been considered to be the main elements of the theory of subjective probability:

a) Non-negativity, Normality, and Additivity
b) Strict or Jeffrey-style conditionalization
c) Reflection Principle
d) Principal Principle

These principles have been formulated in terms of *de dicto* credences. The important question now is, “What happens if we take *de se* credences into consideration?”

Here is my conjecture: The entire theory of subjective probability needs substantial modifications for the proper treatment of *de se*/*de nunc* credences, except the synchronic axioms. Let me outline those modifications: In this dissertation, I have argued that we need to replace SC/JC with GSJC. In his recent paper (2007), Adam Elga offered a new variant of the Reflection Principle for agents who have lost track of what time it is. I agree with his view that the original Reflection Principle needs to be modified to deal with such cases, but I suspect that Elga’s suggestion for that modification is inadequate. Plus, I believe that we need a new variant of the Principal Principle, which will connect objective chance and *de se* credences. To my knowledge, no one previously has mentioned the need to modify the Principal Principle in order to deal properly with *de se* credences.

The theory of *de se* subjective probability is a vast territory consisting of uncharted regions. In this dissertation, I explored one of its toughest parts, but the fun is not over yet. There are still unexplored lands waiting for us.

APPENDIX A

EQUIVALENCE BETWEEN GSJC$^-$ AND GSJC.

(=>) Suppose GSJC$^-$ and show GSJC. Let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be an agent B's general time-observation partition from t_n to t_{n+m}, where $\mathcal{E}_o=\&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})$. By definition, $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is B's general time-observation partition from t_n to t_{n+m} over $[t_0,t_{n+m}]$. Let X be an arbitrary tensed proposition. By GSJC$^-$,

(1) $C_{n+m}(X)=\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$ if $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is optimal and sufficiently inclusive for X,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_\mathrm{o}$ for the m epistemic moments earlier time. For any $o\in O$, $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ is its own abbreviation and $\mathcal{W}^*{}_o$ is the vacuous complement of $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$. Clearly, $C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)=C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o\&\mathcal{R}^*{}_o)$, where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$, $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_o$ for the m epistemic moments earlier time, and $\mathcal{R}^*{}_o$ is the vacuous sequential re-indexicalization of

$\mathcal{W}^*_o$ for the m epistemic moments earlier time.[77] Hence, $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is sufficiently inclusive for X. Therefore,

(2) $C_{n+m}(X)=\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$ if $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is optimal for X.

In other words, GSJC is true. Done.

(<=) Suppose GSJC and show GSJC$^-$. Let $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be the general time-observation partition from t_n to t_{n+m}. Given this partition, let $\{\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p\}_{p\in P}$ be B's general time-observation partition from t_n to t_{n+m} over $[t_{i\leq n},t_{n+m}]$ i.e., for all $p\in P$, $\mathcal{E}_p=\mathcal{E}_o$, $\mathcal{V}_p=\mathcal{V}_o$, and $\mathcal{W}_p=\&_{1\leq k\leq n\text{-}i}(W^k{}_o \text{ at prev}_{m+k\text{-}1})$ for some $i\in\{1,\ldots,n+1\}$. By GSJC,

(3) $C_{n+m}(X)=\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$ if $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is optimal for X,

where $\mathcal{D}_o$ is the sequential de-indexicalization of $\mathcal{E}_o$ under $\mathcal{V}_o$ and $\mathcal{R}_o$ is the sequential re-indexicalization of $\mathcal{W}_\text{o}$ for the m epistemic moments earlier time.

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[77] Since $\mathcal{W}_p$* is the complement of $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ for $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$, $\mathcal{W}_p^*=\&_{n+1\leq k\leq n+1}(W^k{}_p \text{ in } prev_{m+k\text{-}1})=T$, where T is a tautology. By definition, $\mathcal{R}_p$* is the re-indexicalization of $\mathcal{W}_p$*, where $\mathcal{R}_p^*=\&_{n+1\leq k\leq n+1}(W^k{}_p \text{ in } prev_{k\text{-}1})$. Since no k satisfies $n+1\leq k\leq n+1$, $\mathcal{R}_p$* is vacuously true, i.e., $\mathcal{R}_p^*=T$, where T is a tautology.

Assume that $\{\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p\}_{p\in P}$ is optimal and sufficiently inclusive for X i.e., these conditions hold: First, for each $p\in P$, $C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)$ is conditioned upon well-specified temporal information, and the truth-value of X is invariant within $v^m{}_p$ and that of $E^k{}_p$ is invariant within $v^k{}_p$ for any $k\in\{1,\ldots,m\}$. Second, for each $o\in O$ and $p\in P$, if $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ is an abbreviation of $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ and $\mathcal{W}_p*$ is the complement, then $C_n(X$ in $v^m{}_p/\ \mathcal{D}_p\&\mathcal{R}_p)=C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p\&\mathcal{R}*_p)$, where $\mathcal{D}_p$ is the sequential de-indexicalization of $\mathcal{E}_p$ under $\mathcal{V}_p$, and $\mathcal{R}_p$ and $\mathcal{R}_p*$ are the sequential re-indexicalizations of $\mathcal{W}_p$ and $\mathcal{W}_p*$ for the m epistemic moments earlier time. Given these assumptions, it suffices to show

$$(4)\ \ C_{n+m}(X)=\sum\nolimits_{p\in P}C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)C_{n+m}(\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p).$$

To show this, for each $p\in P$, let O_p be the set of $o\in O$ such that $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ is an abbreviation of $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$. Clearly, $\{O_p\}_{p\in P}$ is a partition of O. By this fact and (3),

$$(5)\ \ C_{n+m}(X)=\sum\nolimits_{p\in P}\sum\nolimits_{o\in Op}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o) \text{ if }$$

$$\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O} \text{ is optimal for } X.$$

Consider any $p\in P$. Then, $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ is an abbreviation of $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ for each $o\in O_p$. For each $o\in O_p$, let $\mathcal{W}_o*$ be the complement of $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ for $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$. So for each $o\in O_p$, $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o=\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p\&\mathcal{W}_o*$. On the one hand, $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ is incompatible with $\mathcal{E}_{o'}\&\mathcal{V}_{o'}\&\mathcal{W}_{o'}$ for any $o'\notin O_p$ by its construction. Since $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is a partition, this means that $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ entails $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ for some $o\in O_p$. So $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ entails

$\vee_{o\in Op}\ \mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$. On the other hand, $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ entails $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ as $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o=$ $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p\&\mathcal{W}_o{}^*$. In sum, $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ is equivalent to $\vee_{o\in Op}\ \mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$. Hence,

$$\text{(6)}\quad C_{n+m}(\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p)=\sum_{o\in Op}C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o).$$

Since $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ is an abbreviation of $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ and $\mathcal{W}_o{}^*$ is the complement, $C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)=C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p\&\mathcal{R}^*{}_o)$ by (iv), (where …). Since $\mathcal{E}_p= \mathcal{E}_o$, $\mathcal{D}_p= \mathcal{D}_o$. Since $\mathcal{W}_o= \mathcal{W}_p\&\mathcal{W}^*{}_o$, $\mathcal{R}_o= \mathcal{R}_p\&\mathcal{R}^*{}_o$. Hence,

$$\text{(7)}\quad C_n(X \text{ in } v^m{}_p/\ \mathcal{D}_p\&\mathcal{R}_p)=C_n(X \text{ in } v^m{}_p/\ \mathcal{D}_p\&\mathcal{R}_p\&\mathcal{R}^*{}_o)=C_n(X \text{ in } v^m{}_p/\mathcal{D}_o\&\mathcal{R}_o).$$

Thus,

$$\text{(8)}\quad \text{if } \{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O} \text{ is optimal for } X,\ C_{n+m}(X)=$$

$$\sum_{p\in P}\sum_{o\in Op}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)= \text{(by (5))}$$

$$\sum_{p\in P}C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)\sum_{o\in Op}C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)= \text{(by (7))}$$

$$\sum_{p\in P}C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)C_{n+m}(\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p). \qquad \text{(by (6))}$$

Remember that for all $p\in P$, $\mathcal{E}_p=\mathcal{E}_o$ and $\mathcal{V}_p=\mathcal{V}_o$. By this fact and the given assumptions, for each $o\in O$, $C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)$ is conditioned upon well-specified temporal information, and the truth-value of X is invariant within $v^m{}_o$ and that of $E^k{}_o$ is invariant within $v^k{}_o$ for any $k \in \{1,\ldots,m\}$. In other words, $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ is optimal for X. Done.

APPENDIX B

EQUIVALENCE BETWEEN GSR$^+$ AND GSJC$^+$

Let <Δ,Ω,Φ,Ψ> be the model for *B*'s credences and $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ be her general time-observation partition from t_n to t_{n+m} constructed from Ω and Ψ, where $\mathcal{E}_o$= $\&_{1\leq k\leq m}(E^k{}_o$ at $\mathrm{prev}_{m-k})$, $\mathcal{V}_o$=$\&_{1\leq k\leq m}(V^k{}_o$ at $\mathrm{prev}_{m-k})$, and $\mathcal{W}_o$= $\&_{1\leq k\leq n+1}(W^k{}_o$ at $\mathrm{prev}_{m+k-1})$.

Let $C_{n\backslash}$, $C_{n+m}\in\Delta$ be her credence functions at t_n and t_{n+m}. Then, these facts are provable:

(9) If C_{n+m} is synchronically coherent and related by GSR$^+$ to C_n, then C_{n+m} is also related by GSJC$^+$ to C_n.

(10) If C_n is synchronically coherent and C_{n+m} is related by GSJC$^+$ to C_n, then C_{n+m} is also related by GSR$^+$ to C_n.

To prove (9), suppose that C_{n+m} satisfies the standard axioms and that $C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)$ for each $o\in O$, where $\mathcal{D}_o$= $\&_{1\leq k\leq m}(E^k{}_o$ in $v^k{}_o)$ and $\mathcal{R}_o$=$\&_{1\leq k\leq n+1}(W^k{}_o$ at $\mathrm{prev}_{k-1})$. Then,

(11) $C_{n+m}(X)$=

$\sum_{o\in O}C_{n+m}(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$= (by Additivity)

$$\sum_{o\in O} C_{n+m}(X/\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)= \qquad \text{(def. of } P(\text{-/-})\text{)}$$

$$\sum_{o\in O} C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o). \qquad \text{(by supposition)}$$

Done. Note that I used the assumed synchronic coherence of C_{n+m} in the second line.

To prove (10), suppose that C_n satisfies the standard axioms and that $C_{n+m}(X)=$ $\sum_{o\in O} C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$ for any $X\in\Omega$. So C_n and C_{n+m} are functions mapping Ω into R. (Note that we are not assuming yet that C_{n+m} is synchronically coherent.) For each $o\in O$, construct a function P_o mapping $\Omega_o=_{\text{df}}\{X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o|X\in\Omega\&o\in O\}$ into R as follows:

(12) $\quad P_o(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=_{\text{df}}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o).$

Let T be that 0=0, and F be that 0≠0. Thus, these facts are derivable:

(13) $\quad P_o(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=P_o(\mathrm{T}\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=$

$$C_n(\mathrm{T} \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o).$$

For (T in $v^m{}_o$) is a tautology (e.g., "That 0=0 is true in June 2009" is a tautology).[78]

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[78] A careful reader may complain that if $v^m{}_o$ does not refer to an existing interval, (T in $v^m{}_o$) might be false. For example, "That 0=0 is true in June 2009" might be false if the world, including time itself, ceased to exist at some time in 2008. If such a case is taken into consideration, then I suggest interpreting "X in v" as the tensed proposition that is true if and only if, if v refers to an existing interval of time, then X is true throughout v, and, if v does not refer to any existing interval of time, then X is a tautology.

(14) $P_o(\mathrm{F})=P_o(\mathrm{F}\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=$

$$C_n(\mathrm{F}\text{ in }v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=0.$$

For (F in $v^m{}_o$) is a contradiction (e.g., “That 0≠0 is true in June 2009” is obviously a contradiction). By (12) and (13),

(15) $P_o(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)/P_o(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(X\text{ in }v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)$.

(Also, remember that $P_o(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)>0$ by the definition of the sequential time-observation partition.) Next, we define function P mapping Ω into R as follows:

(16) $P(X)=_{\mathrm{df}}\sum_{o\in O}P_o(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$ for any $X\in\Omega$.

Hence,

(17) $P(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=_{\mathrm{df}}\sum_{o*\in O}P_o((X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)\&(\mathcal{E}_{o*}\&\mathcal{V}_{o*}\&\mathcal{W}_{o*}))$.

(18) $P(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=_{\mathrm{df}}\sum_{o*\in O}P_o((\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)\&(\mathcal{E}_{o*}\&\mathcal{V}_{o*}\&\mathcal{W}_{o*}))$.

Since $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is a partition, $(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)\&(\mathcal{E}_{o*}\&\mathcal{V}_{o*}\&\mathcal{W}_{o*})$ is a contradiction whenever $o\neq o*$, for any o, $o*\in O$. Thus,

$$(19)\qquad P(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=P_o(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o). \qquad \text{(by (17))}$$

$$(20)\qquad P(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=P_o(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o). \qquad \text{(by (18))}$$

So

$$(21)\qquad P(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)/P(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=$$

$$P_o(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)/P_o(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)= \qquad \text{(by (19) and (20))}$$

$$C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o). \qquad \text{(by (15))}$$

By (12), (16), and supposition,

$$(22)\qquad P(X)=\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_{n+m}(X)$$

for any $X\in\Omega$. Hence, $P=C_{n+m}$. By substitution in (21),

$$(23)\qquad C_{n+m}(X\&\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)/C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o).$$

In other words, GSR^+ is true. Done. Note that I depended upon the synchronic coherence of C_n to prove (13) and (14).

APPENDIX C

TRANSLATION BETWEEN TEMPORAL CONTEXTS

Remember that $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ is B's general time-observation partition from t_n to t_{n+m}, $\{\mathcal{F}_p \& \mathcal{V}_p \& \mathcal{W}_p\}_{p\in P}$ is B's sequential time-observation partition from t_{n+m} to t_{n+m+l}, and $\{\mathcal{G}_q \& \mathcal{V}_q \& \mathcal{W}_q\}_{q\in Q}$ be B's sequential time-observation partition from t_n to t_{n+m+l}, where

(24) $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$, $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$, and

$\mathcal{W}_o = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})$,

(25) $\mathcal{F}_p = \&_{1\leq k\leq l}(F^k{}_p \text{ at prev}_{l-k})$, $\mathcal{V}_p = \&_{1\leq k\leq l}(V^k{}_p \text{ at prev}_{l-k})$, and

$\mathcal{W}_p = \&_{1\leq k\leq n+m+1}(W^k{}_p \text{ at prev}_{l+k-1})$, and

(26) $\mathcal{G}_q = \&_{1\leq k\leq m+l}(G^k{}_q \text{ at prev}_{m+l-k})$, $\mathcal{V}_q = \&_{1\leq k\leq m+l}(V^k{}_q \text{ at prev}_{m+l-k})$, and

$\mathcal{W}_q = \&_{1\leq k\leq n+1}(W^k{}_q \text{ at prev}_{m+l+k-1})$.

I claim that

(27) $\mathcal{G}_q = \&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m+l-k}) \& \&_{1\leq k\leq l}(F^k{}_p \text{ at prev}_{l-k})$;

(28) $\mathcal{V}_q = \&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m+l-k}) \& \&_{1\leq k\leq l}(V^k{}_p \text{ at prev}_{l-k})$;

(29) $\mathcal{W}_q = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+l+k-1})$ and $\mathcal{R}_q = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1}) = \mathcal{R}_o$; and

(30) $\mathcal{W}_p = \&_{1\leq k\leq m}(V^k{}_o \text{ at } \text{prev}_{m+l-k}) \& \&_{1\leq k\leq n+1}(W^k{}_o \text{ at } \text{prev}_{m+l+k-1})$ and

$\mathcal{R}_p = \&_{1\leq k\leq m}(V^k{}_o \text{ at } \text{prev}_{m-k}) \& \&_{1\leq k\leq n+1}(W^k{}_o \text{ at } \text{prev}_{m+k-1}) = \mathcal{V}_o \& \mathcal{W}_o$,

	$t_{n+m+l}, \ldots, t_{n+m+l}$	$t_{n+m}, \ldots, t_{n+1}$	$t_n, \ldots, t_0$
Time indexicals used at t_{n+m+l}	$\text{prev}_0, \ldots, \text{prev}_{l-1}$	$\text{prev}_l, \ldots, \text{prev}_{m+l-1}$	$\text{prev}_{m+l}, \ldots, \text{prev}_{n+m+l}$
Time indexicals used at t_{n+m}		$\text{prev}_0, \ldots, \text{prev}_{m-1}$	$\text{prev}_m, \ldots, \text{prev}_{n+m}$
Time indexicals used at t_n			$\text{prev}_0, \ldots, \text{prev}_m$

Figure 14: Referents of Indexicalsin Different Temporal Contexts. Each row shows B's indexical specification of various times, from points of views at different times.

To see why, first, see Figure 14: In this figure, we find temporal moments t_{n+m+l}, ..., t_0, to which we refer by using "t_{n+m+l}," ..., "t_0." However, the given agent, *B*, may not have such non-indexical terms to refer to them with. Even in such a case, *B* can still use indexicals terms to refer to those moments. In doing so, *B* can refer to the same moment(s) by using different time indexicals at different times: For instance, she can refer to the same moment t_n by using "prev_{m+l}" at t_{n+m+l}, by using "prev_m" at t_{n+m}, and by using "prev_0" at t_n. This means that she can ascribe the same tensed propositions to the same moment, by believing different tensed propositions at different moments: For example, let *R* be the tensed proposition that it is raining in White House now. Then, *B* can ascribe *R* to t_n, by believing [*R* at prev_{m+l}] at t_{n+m+l}, by believing [*R* at prev_m] at t_{n+m}, and by believing [*R* at prev_0] at t_n.

	$t_{n+m+l}, \ldots, t_{n+m+1}$	$t_{n+m}, \ldots, t_{n+1}$	$t_n, \ldots, t_0$
Time indexicals used at t_{n+m+l}	$\text{prev}_0, \ldots, \text{prev}_{l-1}$	$\text{prev}_l, \ldots, \text{prev}_{m+l-1}$	$\text{prev}_{m+l}, \ldots, \text{prev}_{n+m+l}$
Observations from t_{n+1} to t_{n+m+l}	$G^{m+l}{}_q, \ldots, G^{m+1}{}_q$	$G^m{}_q, \ldots, G^1{}_q$	
Time intervals from t_0 to t_{n+m+l}	$V^{m+l}{}_q, \ldots, V^{m+1}{}_q$	$V^m{}_q, \ldots, V^1{}_q$	$W^1{}_q, \ldots, W^{n+1}{}_q$

Figure 15: <u>Indexicals, Observations, and Intervals 1</u>.This figure shows how B can ascribe various observations and time intervals to the past epistemic moments.

Second, see Figure 15: Consider any $\mathcal{G}_q\&\mathcal{V}_q\&\mathcal{W}_q$, (where $\mathcal{G}_q=\&_{1\leq k\leq m+l}(G^k{}_q$ at $\text{prev}_{m+l-k})$, $\mathcal{V}_q=\&_{1\leq k\leq m+l}(V^k{}_q$ at $\text{prev}_{m+l-k})$, and $\mathcal{W}_q=\&_{1\leq k\leq n+1}(W^k{}_q$ at $\text{prev}_{m+l+k-1}))$.

Because of its construction, if *B* believes $\mathcal{G}_q\&\mathcal{V}_q\&\mathcal{W}_q$ at t_{n+m+l}, she is ascribing $<G^{m+l}{}_q\&V^{m+l}{}_q, \ldots, G^1{}_q\&V^1{}_q, W^1{}_q, \ldots, W^{n+1}{}_q>$ to $<t_{n+m+l}, \ldots, t_{n+1}, t_n, \ldots, t_0>$ at that moment.[79] Similarly, if she does not fully disbelieve $\mathcal{G}_q\&\mathcal{V}_q\&\mathcal{W}_q$ at t_{n+m+l}, she is not completely ruling out the ascription of $<G^{m+l}{}_q\&V^{m+l}{}_q, \ldots, G^1{}_q\&V^1{}_q, W^1{}_q, \ldots, W^{n+1}{}_q>$ to $<t_{n+m+l}, \ldots, t_{n+1}, t_n, \ldots, t_0>$ at that moment. Since $C_{n+m+l}(\mathcal{G}_q\&\mathcal{V}_q\&\mathcal{W}_q)> 0$, *B* is not fully ruling out that ascription at t_{n+m+l}.

Third, see Figure 16: Suppose, for reductio, that *she* ruled out this ascription of $<G^m{}_q\&V^m{}_q, \ldots, G^1{}_q\&V^1{}_q, W^1{}_q, \ldots, W^{n+1}{}_q>$ to $<t_{n+m}, \ldots, t_{n+1}, t_n, \ldots, t_0>$ at t_{n+m}. If so, she will remember at t_{n+m+l} that she has already ruled it out. In this case, *B* will rule it out at t_{n+m+l}

[79] Of course, I am not assuming that *B* knows at t_{n+m+l} that the moments to which she is ascribing the tensed propositions are $<t_{n+m+l}, \ldots, t_{n+1}, t_n, \ldots, t_0>$. Since she can use indexicals to refer to those epistemic moments, the ability to identify those moments in non-indexical ways is unnecessary for such an ascription.

as well, which contradicts the last paragraph. By reductio, she did not rule out the ascription of $<G^m{}_q\&V^m{}_q, \ldots, G^1{}_q\&V^1{}_q, W^1{}_q, \ldots, W^{n+1}{}_q>$ to $<t_{n+m}, \ldots, t_{n+1}, t_n, \ldots, t_1>$ at t_{n+m+l}.

	$t_{n+m+l}, \ldots, t_{n+m+1}$	$t_{n+m}, \ldots, t_{n+1}$	$t_n, \ldots, t_0$
Time indexicals used at t_{n+m+l}	$\text{prev}_0, \ldots, \text{prev}_{l-1}$	$\text{prev}_l, \ldots, \text{prev}_{m+l-1}$	$\text{prev}_{m+l}, \ldots, \text{prev}_{n+m+l}$
Observations from t_{n+1} to t_{n+m+l}	$G^{m+l}{}_q, \ldots, G^{m+1}{}_q$	$G^m{}_q, \ldots, G^1{}_q$	
Time intervals from t_0 to t_{n+m+l}	$V^{m+l}{}_q, \ldots, V^{m+1}{}_q$	$V^m{}_q, \ldots, V^1{}_q$	$W^1{}_q, \ldots, W^{n+1}{}_q$
Time indexicals used at t_{n+m}		$\text{prev}_0, \ldots, \text{prev}_{m-1}$	$\text{prev}_m, \ldots, \text{prev}_{n+m}$
Observations from t_{n+1} to t_{n+m}		$E^m{}_o, \ldots, E^1{}_o$	
Time intervals from t_0 to t_{n+m}		$V^m{}_o, \ldots, V^1{}_o$	$W^1{}_o, \ldots, W^{n+1}{}_o$

Figure 16: Indexicals, Observations, and Intervals 2. The dark area covers the indexicals, observations, and times which have no counterpart in $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$.

Now, construct $\mathcal{E}^*\&\mathcal{V}^*\&\mathcal{W}^*$ so that $\mathcal{E}^*=\&_{1\le k\le m}(G^k{}_q \text{ at prev}_{m-k})$, $\mathcal{V}^*=\&_{1\le k\le m}(V^k{}_q \text{ at prev}_{m-k})$, and $\mathcal{W}^* =\&_{1\le k\le n+1}(W^k{}_q \text{ at prev}_{m+k-1})$. Due to its construction, if B fully disbelieved $\mathcal{E}^*\&\mathcal{V}^*\&\mathcal{W}^*$ at t_{n+m}, she would've ruled out the ascription of $<G^m{}_q\&V^m{}_q, \ldots, G^1{}_q\&V^1{}_q, W^1{}_q, \ldots, W^{n+1}{}_q>$ to $<t_{n+m}, \ldots, t_{n+1}, t_n, \ldots, t_1>$ at that moment. Since she does not rule out that ascription, B does not fully disbelieve $\mathcal{E}^*\&\mathcal{V}^*\&\mathcal{W}^*$ at t_{n+m}. In other words, $C_{n+m}(\mathcal{E}^*\&\mathcal{V}^*\&\mathcal{W}^*)>0$. However, remember that $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is B's general time-observation partition from t_n to t_{n+m}. By definition, the partition exhausts similarly constructed tensed propositions whose credences at t_{n+m} are strictly positive. Thus, there must be $o\in O$ such that $\mathcal{E}^*\&\mathcal{V}^*\&\mathcal{W}^*=\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$. For this $o\in O$,

(31) $G^m{}_q = E^m{}_o, \ldots, G^1{}_q = E^1{}_o$,

(32) $V^m{}_q = V^m{}_o, \ldots, V^1{}_q = V^1{}_o$, and

(33) $W^1{}_q = W^1{}_o, \ldots, W^{n+1}{}_q = W^{n+1}{}_o$.

Fourth, see Figure 17: B does not rule out the ascription of $<G^{m+l}{}_q \& V^{m+l}{}_q, \ldots, G^{m+1}{}_q \& V^{m+1}{}_q, V^m{}_q, \ldots, V^1{}_q, W^1{}_q, \ldots, W^{n+1}{}_q>$ to $<t_{n+m+l}, \ldots, t_0>$ at t_{n+m+l}. Now, construct $\mathcal{F}{*}\&\mathcal{V}{*}\&\mathcal{W}{*}$ so that $\mathcal{F}{*} = \&_{1\leq k\leq l}(G^{m+k}{}_q$ at $\text{prev}_{l-k})$, $\mathcal{V}{*} = \&_{1\leq k\leq l}(V^{m+k}{}_q$ at $\text{prev}_{l-k})$, and $\mathcal{W}{*} = \&_{1\leq k\leq m}(W^k{}_q$ at $\text{prev}_{l+k-1})\&\&_{1\leq k\leq n+1}(W^k{}_q$ at $\text{prev}_{m+l+k-1})$. Because of its construction, if B fully disbelieves $\mathcal{F}{*}\&\mathcal{V}{*}\&\mathcal{W}{*}$ at t_{n+m+l}, she would completely rule out the ascription of $<G^{m+l}{}_q \& V^{m+l}{}_q, \ldots, G^{m+1}{}_q \& V^{m+1}{}_q, V^m{}_q, \ldots, V^1{}_q, W^1{}_q, \ldots, W^{n+1}{}_q>$ to $<t_{n+m+l}, \ldots, t_0>$ at that moment, which would contradict the above fact. Hence, $C_{n+m+l}(\mathcal{F}{*}\&\mathcal{V}{*}\&\mathcal{W}{*})>0$. However, $\{\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p\}_{p\in P}$ exhausts the similarly constructed tensed propositions whose credences at t_{n+m+l} are strictly positive.

	$t_{n+m+l}, \ldots, t_{n+m+1}$	$t_{n+m}, \ldots, t_{n+1}$	$t_n, \ldots, t_0$
Time indexicals used at t_{n+m+l}	$\text{prev}_0, \ldots, \text{prev}_{l-1}$	$\text{prev}_l, \ldots, \text{prev}_{m+l-1}$	$\text{prev}_{m+l}, \ldots, \text{prev}_{n+m+l}$
Observations from t_{n+1} to t_{n+m+l}	$G^{m+l}{}_q, \ldots, G^{m+1}{}_q$	$G^m{}_q, \ldots, G^1{}_q$	
Time intervals from t_0 to t_{n+m+l}	$V^{m+l}{}_q, \ldots, V^{m+1}{}_q$	$V^m{}_q, \ldots, V^1{}_q$	$W^1{}_q, \ldots, W^{n+1}{}_q$
Observations from t_{n+m+1} to t_{n+m+l}	$F^m{}_p, \ldots, F^1{}_p$		
Time intervals from t_0 to t_{n+m+l}	$V^m{}_p, \ldots, V^1{}_p$	$W^1{}_p, \ldots, W^m{}_p$	$W^{m+1}{}_p, \ldots, W^{n+m+1}{}_p$

Figure 17: Indexicals, Observations, and Intervals 3. The dark area covers the observational data in $\mathcal{G}_q$ that have no counterparts in $\mathcal{F}_p$.

Hence, there exists $p\in P$ such that $\mathcal{F}*\&\mathcal{V}*\&\mathcal{W}*=\mathcal{F}_p\&\mathcal{V}_p\&\mathcal{W}_p$. For this $p\in P$,

(34) $G^{m+l}{}_q=F^l{}_p, \quad \ldots, G^{m+1}{}_q=F^1{}_p,$

(35) $V^{m+l}{}_q=V^l{}_p, \ldots, \; V^{m+1}{}_q=V^1{}_p,$

(36) $V^m{}_q=W^1{}_p, \ldots, \; V^1{}_{q=}W^m{}_p$, and

(37) $W^1{}_q=W^{m+1}{}_p, \ldots, \; W^{n+1}{}_{q=}W^{n+m+1}{}_p.$

Therefore,

(38) $\mathcal{G}_q=\&_{1\leq k\leq m+l}(G^k{}_q \text{ at prev}_{m+l-k})=$ (by (26))

$\&_{1\leq k\leq m}(G^k{}_q \text{ at prev}_{m+l-k})\&\&_{1\leq k\leq l}(G^{m+k}{}_q \text{ at prev}_{l-k})=$ (by definition)

$\&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m+l-k})\&\&_{1\leq k\leq l}(F^k{}_p \text{ at prev}_{l-k}),$ (by (31)&(34))

(39) $\mathcal{V}_q=\&_{1\leq k\leq m+l}(V^k{}_q \text{ at prev}_{m+l-k})=$ (by (26))

$\&_{1\leq k\leq m}(V^k{}_q \text{ at prev}_{m+l-k})\&\&_{1\leq k\leq l}(V^{m+k}{}_q \text{ at prev}_{l-k})=$ (by definition)

$\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m+l-k})\&\&_{1\leq k\leq l}(V^k{}_p \text{ at prev}_{l-k})$, and (by (32)&(35))

(40) $\mathcal{W}_q=\&_{1\leq k\leq n+1}(W^k{}_q \text{ at prev}_{m+l+k-1})=$ (by (26))

$\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+l+k-1}).$ (by (33))

Since $\mathcal{R}_q$ is the sequential re-indexicalization of $\boldsymbol{\mathcal{W}}_q$ for the $m+l$ epistemic moments earlier time and $\mathcal{R}_o$ is the sequential re-indexicalization of $\boldsymbol{\mathcal{W}}_o$ for the m epistemic moments earlier time,

(41) $\mathcal{R}_q = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k\text{-}1}) = \mathcal{R}_o.$ (by definition)

Also,

(42) $\boldsymbol{\mathcal{W}}_p = \&_{1\leq k\leq n+m+1}(W^k{}_p \text{ at prev}_{l+k\text{-}1}) =$ (by (25))

$\&_{1\leq k\leq m}(W^k{}_p \text{ at prev}_{l+k\text{-}1})\&\&_{1\leq k\leq n+1}(W^{m+k}{}_p \text{ at prev}_{m+l+k\text{-}1}) =$ (by definition)

$\&_{1\leq k\leq m}(V^k{}_q \text{ at prev}_{m+l\text{-}k})\&\&_{1\leq k\leq n+1}(W^k{}_p \text{ at prev}_{m+l+k\text{-}1}) =$ (by (36)&(37))

$\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m+l\text{-}k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+l+k\text{-}1}).$ (by (32)&(33))

Since $\mathcal{R}_p$ is the sequential re-indexicalization of $\boldsymbol{\mathcal{W}}_p$ for the l epistemic moments earlier time,

(43) $\mathcal{R}_p = \&_{1\leq k\leq n+m+1}(W^k{}_p \text{ at prev}_{k\text{-}1}) =$ (by (25))

$\&_{1\leq k\leq m}(W^k{}_p \text{ at prev}_{k\text{-}1})\&\&_{1\leq k\leq n+1}(W^{m+k}{}_p \text{ at prev}_{m+k\text{-}1}) =$ (by definition)

$\&_{1\leq k\leq m}(V^k{}_q \text{ at prev}_{m\text{-}k})\&\&_{1\leq k\leq n+1}(W^k{}_q \text{ at prev}_{m+k\text{-}1})=$ (by (36)&(37))

$\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m\text{-}k})\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k\text{-}1})=$ (by (32)&(33))

$\boldsymbol{v}_o\&\boldsymbol{w}_o.$ (by (24))

Done.

APPENDIX D

EQUIVALENCE BETWEEN GSJC$^+$ AND GSJC^0

Let C_n, $C_{n+m} \in \Delta$ be B's credence functions at t_n and t_{n+m}. Then, these facts are provable:

(44) If C_{n+m} is related by GSJC^0 to C_n, then C_{n+m} is related by GSJC$^+$ to C_n, and

(45) If C_n and C_{n+m} are synchronically coherent and C_{n+m} is related by GSJC$^+$ to C_n, then C_{n+m} is also related by GSJC^0 to C_n.

It is relatively easy to prove (44): Let $<\Delta,\Omega,\Phi,\Psi>$ be a model for an agent B's credences and $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ be her general time-observation partition from t_n to t_{n+m} constructed from Ω and Ψ, where $\mathcal{E}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ at prev}_{m-k})$, $\mathcal{V}_o = \&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$, and $\mathcal{W}_o = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})$. Suppose that C_{n+m} is related by GSJC^0 to C_n, and show that C_{n+m} is related by GSJC$^+$ to C_n. Hence, we want to show that $C_{n+m}(X) = \sum_{o\in O} C_n(X \text{ in } v^m{}_o / \mathcal{D}_o \& \mathcal{R}_o) C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)$ for any $X \in \Omega$, where $\mathcal{D}_o = \&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)$ and $\mathcal{R}_o = \&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1})$. Since $\Psi \subseteq \Xi$, $\{\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o\}_{o\in O}$ was constructed from Ω and Ξ. By supposition, $C_{n+m}(X) = \sum_{o\in O} C_n(X \text{ in } v^m{}_o / \mathcal{D}_o \& \mathcal{R}_o)$ $C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)$ if (i) the truth-value of X is invariant within $v^m{}_o$ and that of $E^k{}_o$ is invariant within $v^k{}_o$ for each $k\in\{1,...,m\}$ and (ii) $C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)$ is conditioned upon a well-specified temporal information, for each $o\in O$. Since $X\in\Omega$ and $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ was constructed from Ω and Ψ, (i) and (ii) are satisfied. Done.

It is more difficult to prove (45): Let $<\Delta,\Omega,\Theta,\Xi>$ be an extension of $<\Delta,\Omega,\Phi,\Psi>$ and $\{\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p\}_{p\in P}$ be B's general time-observation partition from t_n to t_{n+m} constructed from Ω and Ξ, where $\mathcal{E}_p=\&_{1\leq k\leq m}(E^k{}_p \text{ at prev}_{m-k})$, $\mathcal{V}_p=\&_{1\leq k\leq m}(V^k{}_p \text{ at prev}_{m-k})$, and $\mathcal{W}_p=\&_{1\leq k\leq n+1}(W^k{}_p \text{ at prev}_{m+k-1})$. Suppose that C_n and C_{n+m} are synchronically coherent and C_{n+m} is related by GSJC$^+$ to C_n, and show that C_{n+m} is also related by GSJC^0 to C_n. To show this, let X be any member of Ω. It suffices to assume the satisfaction of (i) and (ii) and show that $C_{n+m}(X)=\sum_{p\in P}C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)\ C_{n+m}(\ \mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p)$, where $\mathcal{D}_p= \&_{1\leq k\leq m}(E^k{}_p \text{ in } v^k{}_p)$ and $\mathcal{R}_p=\&_{1\leq k\leq n+1}(W^k{}_p \text{ at prev}_{k-1})$. By construction, there exists $\Psi_{<k,p>}\subseteq\Psi$ such that $V^k{}_p\equiv\vee\Psi_{<k,p>}$ for each $k\in\{1,...,m\}$ and $p\in P$ and there also exists $\Psi*_{<k,p>}\subseteq\Psi$ such that $W^k{}_p\equiv\vee\Psi*_{<k,p>}$ for each $k\in\{1,...,n+1\}$ and $p\in P$. For each $p\in P$, construct $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O*p}$ such that $\mathcal{E}_o=\mathcal{E}_p$, $\mathcal{V}_o=\&_{1\leq k\leq m}(V^k{}_o \text{ at prev}_{m-k})$ for some $<V^1{}_o,...,V^m{}_o>\in\Psi_{<1,p>}\times...\times\Psi_{<m,p>}$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{m+k-1})$ for some $<W^1{}_o,...,W^{n+1}{}_o>\in\Psi*_{<1,p>}\times...\times\Psi*_{<n+1,p>}$. Clearly, $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O*p}$ is a partition.

Consider arbitrary $p\in P$. On the one hand, $\vee_{o\in O*p}\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ entails $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$. To see this fact, it suffices to show that $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ entails $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ for any $o\in O*_p$.

So consider any $o\in O^*_p$. First, $\mathcal{E}_o$ clearly entails $\mathcal{E}_p$. Second, $\mathcal{V}_o$ entails $\mathcal{V}_p$. Why? For each $k\in\{1,...,m\}$, (V^k_o at prev$_{m\text{-}k}$) entails (V^k_o at prev$_{m\text{-}k}$), because $V^k_o\in\Psi_{<k,p>}$ and so V^k_o entails $\vee\Psi_{<k,p>}$, which is the same as V^k_p. It clearly follows that $\&_{1\leq k\leq m}$(V^k_o at prev$_{m\text{-}k}$) entails $\&_{1\leq k\leq m}$(V^k_p at prev$_{m\text{-}k}$). Third, $\mathcal{W}_o$ entails $\mathcal{W}_o$. Why? For each $k\in\{1,...,n+1\}$, (W^k_o at prev$_{m+k\text{-}1}$) entails (W^k_o at prev$_{m+k\text{-}1}$), because $W^k_o\in\Psi*_{<k,p>}$ and so W^k_o entails $\vee\Psi*_{<k,p>}$, which is the same as W^k_p. It clearly follows that $\&_{1\leq k\leq n+1}$(W^k_o at prev$_{m+k\text{-}1}$) entails $\&_{1\leq k\leq n+1}$(W^k_p at prev$_{m+k\text{-}1}$).

On the other hand, $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ entails $\vee_{o\in O^*p}\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$: By the construction of $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in Op}$, there exists some $o\in O^*_p$ such that $\mathcal{E}_o=\mathcal{E}_p$, $\mathcal{V}_o=\&_{1\leq k\leq m}$($V^k_o$ at prev$_{m\text{-}k}$) for some $<V^1_o,...,V^m_o>\in\Psi_{<1,p>}\times...\times\Psi_{<m,p>}$, and $\mathcal{W}_o=\&_{1\leq k\leq n+1}$($W^k_o$ at prev$_{m+k\text{-}1}$) for some $<W^1_o,...,W^{n+1}_o>\in\Psi*_{<1,p>}\times...\times\Psi*_{<n+1,p>}$. So V^k_o entails $\vee\Psi_{<k,p>}$ for each $k\in\{1,...,m\}$. Similarly, W^k_o entails $\vee\Psi*_{<k,p>}$ for each $k\in\{1,...,n+1\}$. Since $V^k_p\equiv\vee\Psi_{<k,p>}$ and $W^k_p\equiv\vee\Psi*_{<k,p>}$, V^k_o entails V^k_p for each $k\in\{1,...,m\}$ and W^k_o entails W^k_p for each $k\in\{1,...,n+1\}$. Clearly, (V^k_o at prev$_{m\text{-}k}$) entails (V^k_p at prev$_{m\text{-}k}$) for each $k\in\{1,...,m\}$ and (W^k_o at prev$_{m+k\text{-}1}$) entails (W^k_p at prev$_{m+k\text{-}1}$) for each $k\in\{1,...,n+1\}$. Thus, $\mathcal{V}_p\&\mathcal{W}_p$ entails $\mathcal{V}_o\&\mathcal{W}_o$. Since $\mathcal{E}_p$ clearly entails $\mathcal{E}_o$, $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ entails that $\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$ for some $o\in O^*_p$. Hence, $\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p$ entails $\vee_{o\in O^*p}\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o$.

Since p was arbitrarily chosen from P, $\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p$ is equivalent to $\bigvee_{o\in O^*p}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)$ for each $p\in P$. For each $p\in P$, construct $O_p \subseteq O^*_p$ such that $C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)>0$ for any $o\in O_p$. Define O to be $\bigcup_{p\in P} O_p$. Hence,

(46) $C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)>0$ for any $o\in O$,

(47) $\sum_{o\in O} C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)=$

$\sum_{p\in P}\sum_{o\in Op} C_{n+m}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o)=$ (by the construction of O)

$\sum_{p\in P} C_{n+m}(\bigvee_{o\in Op}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o))=$ (by Additivity)

$\sum_{p\in P} C_{n+m}(\bigvee_{o\in O^*p}(\mathcal{E}_o \& \mathcal{V}_o \& \mathcal{W}_o))=$ (by the construction of each O_p)

$\sum_{p\in P} C_{n+m}(\mathcal{E}_p \& \mathcal{V}_p \& \mathcal{W}_p)=1$, and (by the above equivalence)

(48) $\sum_{o\in O} C_n(\mathcal{D}_o \& \mathcal{R}_o)>0.$ (by (46))

To understand how (48) derives from (46), suppose, for reductio, that $\sum_{o\in O} C_n(\mathcal{D}_o \& \mathcal{R}_o)=0$. Then, $C_n(\mathcal{D}_o \& \mathcal{R}_o)=0$ for any $o\in O$. It means that at t_n, B completely rules out the possibility that [$E^1{}_o$ will be true in $v^1{}_o$,$E^2{}_o$ will be true in $v^2{}_o$, …, $E^m{}_o$ will be true in $v^1{}_o$] and [it is $w^1{}_{\boldsymbol{o}}$ now, it was $w^1{}_{\boldsymbol{o}}$ at the one epistemic moment earlier time, …, it was $w^{n+1}{}_{\boldsymbol{o}}$ at the n+1 epistemic moment earlier time]. At t_{n+m}, B remembers that she ruled out this possibility at the m epistemic moments earlier time. Acknowledging that m epistemic moments have passed, she will rule out the possibility that [$E^1{}_o$&$V^1{}_o$ was true, $E^2{}_o$&$V^2{}_o$ was true, …, $E^m{}_o$&$V^m{}_o$ is true now] and [it was $w^1{}_{\boldsymbol{o}}$ at the m epistemic earlier time, it was $w^1{}_{\boldsymbol{o}}$ at the m+1 epistemic moment earlier time, …, it was $w^{n+1}{}_{\boldsymbol{o}}$ at the m+n+1 epistemic moment earlier time]. So $C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)= 0$. However, this contradicts (46). By reductio, $\sum_{o\in O}C_n(\mathcal{D}_o\&\mathcal{R}_o)>0$. From (46)-(48), it follows that $\{\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o\}_{o\in O}$ is B's sequential time-observation partition from t_n to t_{n+m} over $[t_0,t_{n+m}]$ constructed from Ω and Ψ. Since we supposed that C_{n+m} is related by GSJC$^+$ to C_n,

(49) $$C_{n+m}(X)=\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o).$$

Focus upon the value of $C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)$. For any $o\in O$ such that $o\in O_p$,

(50) $$C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=$$

$$\sum_{p\in P}\sum_{o\in Op}C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)= \qquad \text{(by the construction of } O\text{)}$$

$$\sum_{p\in P}\sum_{o\in O^*p}C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)= \qquad \text{(by the construction of } O_p\text{)}$$

$$\sum_{p\in P}C_{n+m}(\vee_{o\in O^*p}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o))= \qquad \text{(by Additivity)}$$

$$\sum_{p\in P}C_{n+m}(\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p). \qquad (\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p\equiv \vee_{o\in O^*p}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o))$$

Focus upon the value of $C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)$. For any $p\in P$ and $o\in O_p$,

(51) $C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)=$

$C_n(X \text{ in } v^m{}_p/\&_{1\leq k\leq m}(E^k{}_p \text{ in } v^k{}_p)\&\&_{1\leq k\leq n+1}(W^k{}_p \text{ at prev}_{k-1}))=$ (by definition)

$C_n(X \text{ in } v^m{}_o/\&_{1\leq k\leq m}(E^k{}_p \text{ in } v^k{}_p)\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1}))$.

For $C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)$ was assumed to be conditioned upon well-specified temporal description and $w^k{}_o\subseteq w^k{}_p$, for each $k\in\{1,...,n+1\}$. For any $p\in P$ and $o\in O_p$,

(52) $C_n(X \text{ in } v^m{}_o/\&_{1\leq k\leq m}(E^k{}_p \text{ in } v^k{}_p)\&\&_{1\leq k\leq n+1}(W^k{}_o \text{ at prev}_{k-1}))=$

$C_n(X \text{ in } v^m{}_p/\&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_p)\&\&_{1\leq k\leq n+1}(W^k{}_p \text{ at prev}_{k-1}))=$

$C_n(X \text{ in } v^m{}_o/\&_{1\leq k\leq m}(E^k{}_o \text{ in } v^k{}_o)\&\&_{1\leq k\leq n+1}(W^k{}_p \text{ at prev}_{k-1}))$,

because the truth-value of X is invariant within $v^m{}_p$ and that of $E^k{}_o$ $(=E^k{}_p)$ is invariant within $v^k{}_p$, and $v^m{}_o\subseteq v^m{}_p$ and $v^k{}_o\subseteq v^k{}_p$, for any $k\in\{1,...,m\}$. Hence,

(53) $C_{n+m}(X)=$

$\sum_{o\in O}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=$ (by (49))

$\sum_{o\in Op\&p\in P}C_n(X \text{ in } v^m{}_o/\mathcal{D}_o\&\mathcal{R}_o)C_{n+m}(\mathcal{E}_o\&\mathcal{V}_o\&\mathcal{W}_o)=$ (by construction of O)

$\sum_{o\in Op\&p\in P}C_n(X \text{ in } v^m{}_p/\mathcal{D}_p\&\mathcal{R}_p)C_{n+m}(\mathcal{E}_p\&\mathcal{V}_p\&\mathcal{W}_p)=$ (by (50)-(52))

$$\sum_{p\in P} C_n(X \text{ in } v^m{}_p/\boldsymbol{\mathcal{D}}_p\&\boldsymbol{\mathcal{R}}_p)C_{n+m}(\boldsymbol{\mathcal{E}}_p\&\boldsymbol{\mathcal{V}}_p\&\boldsymbol{\mathcal{W}}_p). \qquad \text{(simplification)}$$

Therefore, C_{n+m} is related by GSJC^{+} to C_n. Done.

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